Simple Hypotheses

Homework 5

Problems

1. Returning to the monsoon rain example, consider the gamma density

$$f_X(x|\beta) = \sqrt{\frac{\beta}{\pi x}}\, e^{-\beta x}, \quad x \geq 0.$$

   (a) For n independent observations, $\mathbf{X} = (X_1, \ldots, X_n)$, give the likelihood in terms of the minimal sufficient statistic $T = T(\mathbf{X})$.

   (b) What is the distribution of T?

   (c) Consider the following simple hypothesis on monsoon rains in millimeters.

$$H_0 : \beta = \beta_0 \quad \text{versus} \quad H_1 : \beta = \beta_1$$

   with $\beta_0 > \beta_1$. Find the critical region for the test for $n = 17$ and $\beta_0 = 0.06$ and significance levels $\alpha = 0.05$.

2. Consider the following simple hypothesis on human body temperature in degrees Fahrenheit.

$$H_0 : \mu = 98.6 \quad \text{versus} \quad H_1 : \mu = 98.3.$$

   Assume that the standard deviation of the temperature is $\sigma = 0.68$ degrees Fahrenheit.

   (a) If the distribution of body temperatures is normally distributed, give the critical regions for a $\alpha = 0.05, 0.02, 0.01$ test.

   (b) What happens to the critical region as α decreases? Explain why.

   (c) For 52 randomly chosen healthy adults is measured with the following summary of the data:

$$n = 52, \quad \bar{x} = 98.4.$$

   What decision do you make for each significance level α

   (d) For the case $\alpha = 0.05$, find the power of the test for $n = 52$ and indicate this value using the cutoff value for the test and drawing the sample distribution for the null and alternative hypothesis.

Challenging Problems

1. For the monsoon rain example, take $\beta_1 = 0.04$ and give he cutoff values for the sufficient statistics in the cases $\pi\{0\} = 0.1, 0.2$, the prior probability for H_0 and a relative loss $\ell_{II}/\ell_I = 2$.

2. For the human body temperature example, draw the receiving operator characteristic for alternatives $\mu = 98.3, 98.4$ and 98.5. Which curve is the lowest? Explain your answer.