Confidence Intervals

Homework 9

Problems

1. The focal length of a lens in your lab has been mislabeled. You decide to make repeated measurements of the distance to the image and the object in order to estimate the focal length. Here r_1 is the distance from the lens to the object and r_2 is the distance from the lens to the real image of the object. Here are the measurements.

```
> r1<-c(10.04,9.85,10.11,9.96,10.06,10.03,10.10,10.06,10.09,9.95)
> r2<-c(7.80,7.78,7.71,7.88,7.70,7.81,7.65,8.18,7.67,8.14,8.52,7.64,8.38,8.00,
7.94,7.82,8.09,7.59,8.33,8.40,8.94,7.89,8.89,7.81,7.97)
```

   (a) Give a 98% confidence interval for r_1 and for r_2.

   (b) The focal length f is determined by using the thin lens formula,

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{f}.$$

   Give an estimate $\hat{f}$ based on these measurements and the thin lens formula.

   (c) Use the delta method to give an estimate for standard deviation of $\hat{f}$.

   (d) Use the normal distribution to devise a 95% confidence interval for f.

   (e) Use a bootstrap based on resampling 10,000 times to find a 95% confidence interval. Compare this to the interval given by the delta method.

   (f) Indicate this confidence interval using a histogram of the bootstrap samples.

2. For Major League Baseball, we will take the prior probability of percentage wins as a $Beta(30, 30)$

   (a) Give the 0.025 and the 0.975 quantile for this distribution.

   (b) At the All Star break, three times have the following records

team	wins	losses
1	50	34
2	42	42
3	28	56

   Find a 95% equal tailed credible interval for these three teams.

3. We are looking at the change in density in traffic during the time of "social distancing". The number of cars that pass through an intersection at in the minute after 5PM is modeled as independent Poisson random variables with parameter λ. We take a prior distribution that is $\Gamma(1/2, 3)$

(a) For data $x_1, \ldots, x_n$, describe the posterior distribution of λ.

(b) Using the data

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> cars
 [1]  7  9  6  9 11  4  9  3  4 12  3  4
```

Find 95% credible intervals, one with equal tail probabilities and one that is the high density credible interval.