

# Method of Moments

## Worksheet 15\*

1. Assume that  $X$ , the value an unfair die, has the following probability mass function.

$x$	1	2	3	4	5	6
$f_X(x \alpha)$	$\frac{1}{6}(1-3\alpha)$	$\frac{1}{6}(1-2\alpha)$	$\frac{1}{6}(1-\alpha)$	$\frac{1}{6}(1+\alpha)$	$\frac{1}{6}(1+2\alpha)$	$\frac{1}{6}(1+3\alpha)$

- (a) What values for  $\alpha$  are possible?
- (b) Find the mean  $\mu = E_\alpha X$  and the variance  $\sigma^2 = \text{Var}_\alpha(X)$ .
- (c) Let  $\bar{X}$  be the average of 100 dice rolls find the mean  $E_\alpha \bar{X}$  and the variance  $\text{Var}_\alpha(\bar{X})$ .
- (d) For independent observations,  $\mathbf{x} = (x_1, x_2, \dots, x_{100})$ , of this random variable from a given value of  $\alpha$ , give  $\hat{\alpha}$ , the method of moments estimate.
- (e) Find  $E_\alpha \hat{\alpha}(X_1, \dots, X_{100})$ . Is this estimator biased?
- (f) Provide 10,000 simulations for the case  $\alpha = -1/10$ . Find the mean and standard deviation of these simulations. How does this compare to your results in parts (e)?
- (g) Make a histogram of the values for  $\hat{\alpha}$  and provide a description of the histogram.