Method of Moments*

Worksheet 16

1. Consider the density function for a random variable X,

$$f_X(x|\theta) = \begin{cases} c(\theta)(1+\theta x) & \text{for } -1 \le x \le 1. \\ 0 & \text{otherwise.} \end{cases}$$

   (a) What are the possible values for the parameter θ? Explain your answer.

   (b) What value of $c(\theta)$ is necessary for $f_X(x|\theta)$ to be a valid density function?

   (c) Determine the cumulative distribution function.

   (d) Find the probability, $P_\theta\{X > 1/2\}$. Sketch this probability as the area under the density function for the case $\theta = 1/2$.

   (e) Find the mean $E_\theta X$.

   (f) Use this to find a method of moments estimator $\hat{\theta}$ for θ.

2. Daily rainfall data, in millimeters, is modeled as having a $\Gamma(1/2, \beta)$ distribution. The density is

$$f_X(x|\alpha,\beta) = \begin{cases} 0 & \text{for } x \le 0, \\ \frac{\beta^{1/2}}{\sqrt{\pi}} x^{-1/2} e^{-\beta x} & \text{for } x > 0. \end{cases}$$

   (a) Find the method of moments estimator for β based on rainfall amounts $x_1, x_2, \ldots, x_n$

   (b) Give the estimate $\hat{\beta}$ for the monsoon rainfall amounts in millimeters during July and August, 2017 for Tucson, Arizona.

   3 15 1 37 5 1 8 11 6 9 12 35 22 3 38 1 2

   (c) On a single plot, give both the empirical cumulative distribution function for the data above and the appropriate gamma distribution function.

   (d) Use $\hat{\beta}$ and the gamma distribution commands in R to estimate the probability that a monsoon rain exceeds 25 mm. Indicate this value on the plot.

   (e) Is this estimator biased? How did you reach this conclusion.

   (f) Use the delta method to estimate the variance in the estimator using the value obtained for $\hat{\beta}$ for the $n = 17$ monsoon rainfall amounts.