Topic 18: Composite Hypotheses*

November, 2011

Simple hypotheses limit us to a decision between one of two possible states of nature. This limitation does not allow us, under the procedures of hypothesis testing to address the basic question:

> *Does the length, the reaction rate, the fraction displaying a particular behavior or having a particular opinion, the temperature, the kinetic energy, the Michaelis constant, the speed of light, mutation rate, the melting point, the probability that the dominant allele is expressed, the elasticity, the force, the mass, the parameter value θ_0 increase, decrease or change at all under under a different experimental condition?*

This leads us to consider **composite hypothesis**. In this case, the parameter space Θ is divided into two disjoint regions, Θ_0 and Θ_1. The hypothesis test is now written

$$H_0 : \theta \in \Theta_0 \quad \text{versus} \quad H_1 : \theta \in \Theta_1.$$

Again, H_0 is called the **null hypothesis** and H_1 the **alternative hypothesis**.

For the three alternatives to the question posed above,

- increase would lead to the choices $\Theta_0 = \{\theta; \theta \le \theta_0\}$ and $\Theta_1 = \{\theta; \theta > \theta_0\}$,
- decrease would lead to the choices $\Theta_0 = \{\theta; \theta \ge \theta_0\}$ and $\Theta_1 = \{\theta; \theta < \theta_0\}$, and
- change would lead to the choices $\Theta_0 = \{\theta_0\}$ and $\Theta_1 = \{\theta; \theta \ne \theta_0\}$

for some choice of parameter value θ_0. The effect that we are meant to show, here the nature of the change, is contained in Θ_1. The first two options given above are called **one-sided tests**. The third is called a **two-sided test**,

Rejection and failure to reject the null hypothesis, critical regions, C, and type I and type II errors have the same meaning for a composite hypotheses as it does with a simple hypothesis. Singificance level and power will necessitate an extention of the ideas for smple hypotheses.

1 Power

Power is now a function of the parameter value θ. If our test is to reject H_0 whenever the data fall in a **critical region** C, then the **power function** is defined as

$$\pi(\theta) = P_\theta\{X \in C\}.$$

that gives the probability of rejecting the null hypothesis for a given value of the parameter. Consequently, the ideal power function has

$$\pi(\theta) \approx 0 \text{ for all } \theta \in \Theta_0 \text{ and } \pi(\theta) \approx 1 \text{ for all } \theta \in \Theta_1$$

With this property for the power function, we would rarely reject the null hypothesis when it is true and rarely fail to reject the null hypothesis when it is false. Such a power function shows that we reach the correct decision with probability nearly 1.

In reality, incorrect decisions are made. Thus, for $\theta \in \Theta_0$,

$$\pi(\theta) \text{ is the probability of making a type I error,}$$

i. e., rejecting the null hypothesis when it is indeed true, For $\theta \in \Theta_1$,

$$1 - \pi(\theta) \text{ is the probability of making a type II error,}$$

i.e., failing to reject the null hypothesis when it is false.

The goal is to make the chance for error small. The traditional method is analogous to that employed in the Neyman-Pearson lemma. Fix a **(significance) level** α, now defined to be the largest value of $\pi(\theta)$ in the region Θ_0 defined by the null hypothesis. In other words, by focusing on the value of the parameter in Θ_0 that is most likely to result in an error, we insure that the probability of a type I error is no more that α irrespective of the value for $\theta \in \Theta_0$. Then, we look for a critical region that makes the power function as large as possible for values of the parameter $\theta \in \Theta_1$

Example 1. *Let $X_1, X_2, \ldots, X_n$ be independent $N(\mu, \sigma_0^2)$ random variables with σ_0^2 known and μ unknown. For the composite hypothesis for the* **one-sided test**

$$H_0 : \mu \leq \mu_0 \quad \text{versus} \quad H_1 : \mu > \mu_0.$$

We use the test statistic from the likelihood ratio test and reject H_0 if $\bar{X}$ is too large. Thus, the critical region

$$C = \{\mathbf{x}; \bar{x} \geq k(\mu_0)\}.$$

If μ is the **true mean**, *then the power function*

$$\pi(\mu) = P_\mu\{X \in C\} = P_\mu\{\bar{X} \geq k(\mu_0)\}.$$

As we shall see soon, the value of $k(\mu_0)$ depends on the level of the test.

Note that $\pi(\mu)$ increases with μ. As the actual mean μ increases, then the probability that the sample mean $\bar{X}$ exceeds a particular value $k(\mu_0)$ also increases. To obtain level α for the hypothesis test, we use that fact that $\pi(\mu)$ increases with μ to conclude that the maximum value of π on the set $\Theta_0 = \{\mu; \mu \leq \mu_0\}$ takes place for the value μ_0, i.e.,

$$\alpha = \pi(\mu_0) = P_{\mu_0}\{\bar{X} \geq k(\mu_0)\}.$$

We now use this to find the value $k(\mu_0)$. When μ_0 is the value of the mean, we standardize to give a standard normal random variable

$$Z = \frac{\bar{X} - \mu_0}{\sigma_0/\sqrt{n}}.$$

Choose z_α so that $P\{Z \geq z_\alpha\} = \alpha$. In this case, $\Phi(z_\alpha) = 1 - \alpha$ where Φ is the distribution function for the standard normal, thus

$$P_{\mu_0}\{Z \geq z_\alpha\} = P_{\mu_0}\{\bar{X} \geq \mu_0 + \frac{\sigma_0}{\sqrt{n}} z_\alpha\}$$

and $k(\mu_0) = \mu_0 + (\sigma_0/\sqrt{n}) z_\alpha$.

If μ is the true state of nature, then

$$Z = \frac{\bar{X} - \mu}{\sigma_0/\sqrt{n}}$$

is a standard normal random variable. We use this fact to determine the power function for this test.

$$\pi(\mu) = P_\mu\{\bar{X} \geq \frac{\sigma_0}{\sqrt{n}} z_\alpha + \mu_0\} = P_\mu\{\bar{X} - \mu \geq \frac{\sigma_0}{\sqrt{n}} z_\alpha - (\mu - \mu_0)\} \tag{1}$$

$$= P_\mu\left\{\frac{\bar{X} - \mu}{\sigma_0/\sqrt{n}} \geq z_\alpha - \frac{\mu - \mu_0}{\sigma_0/\sqrt{n}}\right\} = 1 - \Phi\left(z_\alpha - \frac{\mu - \mu_0}{\sigma_0/\sqrt{n}}\right) \tag{2}$$

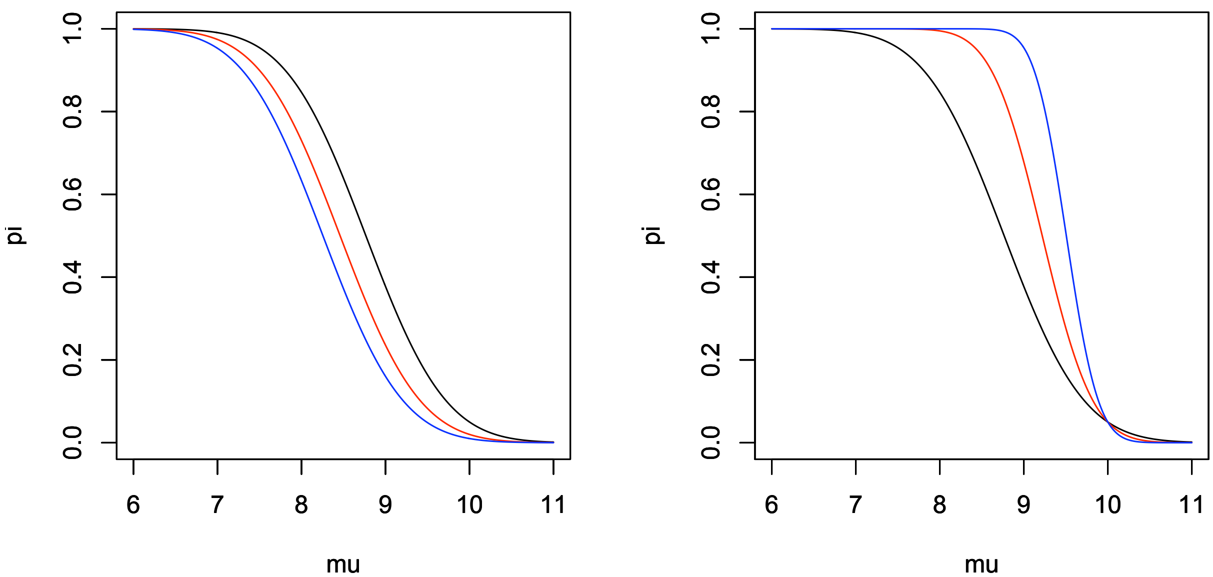

Figure 1: Power function for the one-sided test with alternative "less than". $\mu_0 = 10$, $\sigma_0 = 3$. Note, as argued in the text that π is a decreasing function. **(left)** $n = 16$, $\alpha = 0.05$ (black), 0.02 (red), and 0.01 (blue). Notice that lowering significance level α reduces power $\pi(\mu)$ for each value of μ. **(right)** $\alpha = 0.05$, $n = 15$ (black), 40 (red), and 100 (blue). Notice that increasing sample size n increases power $\pi(\mu)$ for each value of $\mu \leq \mu_0$.

Exercise 2. *If the alternative is less than, show that*

$$\pi(\mu) = \Phi\left(-z_\alpha - \frac{\mu - \mu_0}{\sigma_0/\sqrt{n}}\right).$$

Returning to the example with a model species and its mimic. For the plot of the power function for $\mu_0 = 10$, $\sigma_0 = 3$, and $n = 16$ observations,

```
> zalpha<-qnorm(0.95)
> mu0<-10
> sigma0<-3
> mu<-(600:1100)/100
> n<-16
> z<--zalpha - (mu-mu0)/(sigma0/sqrt(n))
> pi<-pnorm(z)
> plot(mu,pi,type="l")
```

In Figure 1, we vary the values of the significance level α and the values of n, the number of observations in the graph of the power function π

Example 3 (mark and recapture)**.** *We may use mark and recapture to see if a population has reached a dangerously low level. The variables in mark and recapture are*

- *t be the number captured and tagged,*
- *k be the number in the second capture,*
- *r the the number in the second capture that are tagged, and let*

- *N be the total population.*

If N_0 is the level that a wildlife biologist say is dangerously low, then the natural hypothesis is one-sided.

$$H_0 : N \geq N_0 \quad \text{versus} \quad H_1 : N < N_0.$$

The data are the he number in the second capture that are tagged, r. The likelihood function for N is the hypergeometric distribution.

$$L(N|r) = \frac{\binom{t}{r}\binom{N-t}{k-r}}{\binom{N}{k}}$$

and the maximum likelihood estimate is $\hat{N} = [tk/r]$. Thus, higher values for r lead us to lower estimates for N. Let R be the (random) number in the second capture that are tagged, then, for an α level test, we look for the minimum value r_α so that

$$P_N\{R \geq r_\alpha\} \leq \alpha \text{ for all } N \geq N_0. \tag{3}$$

As N increases, then recaptures become less likely and the probability in (3) decreases. Thus, we should set the value of r_α according to the parameter value N_0, the minimum value under the null hypothesis. Let's determine r_α for several values of α using the example from the topic, Maximum Likelihood Estimation, and consider the case in which the critical population is $N_0 = 2000$.

```
> N0<-2000
> t<-200
> k<-400
> alpha<-c(0.05,0.02,0.01)
> ralpha<-qhyper(1-alpha,t,N0-t,k)
> data.frame(alpha,ralpha)
  alpha ralpha
1  0.05     49
2  0.02     51
3  0.01     53
```

For example, we must capture al least 49 that were tagged in order to reject H_0 at the $\alpha = 0.05$ level. In this case the estimate for N is $\hat{N} = [kt/r_\alpha] = 1632$. As anticipated, r_α increases and the critical regions shrinks as the value of α decreases.

Using the level r_α determined using the value N_0 for N, we see that the power function

$$\pi(N) = P_N\{R \geq r_\alpha\}.$$

R is a hypergeometric random variable with mass function

$$f_R(r) = P_N\{R = r\} = \frac{\binom{t}{r}\binom{N-t}{k-r}}{\binom{N}{k}}.$$

The plot for the case $\alpha = 0.05$ is given using the R *commands*

```
> N<-c(1300:2100)
> pi<-1-phyper(49,t,N-t,k)
> plot(N,pi,type="l",ylim=c(0,1))
```

We can increase power by increasing the size of k, the number the value in the second capture. This increases the value of r_α. For $\alpha = 0.05$, we have the table.

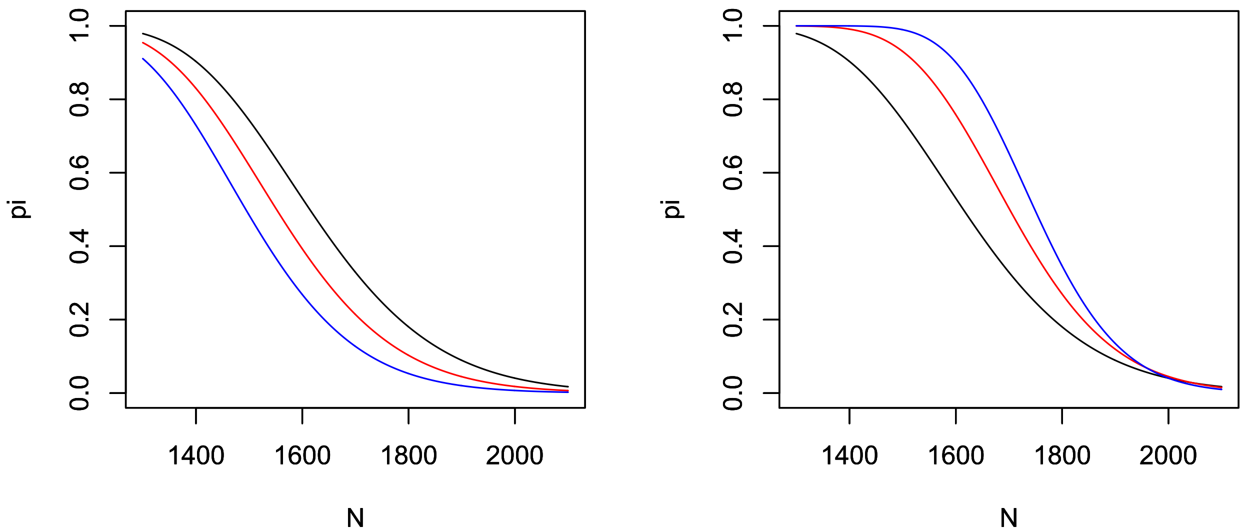

Figure 2: Power function for Lincoln-Peterson mark and recapture test for population $N_0 = 2000$ and $t = 200$ captured and tagged. **(left)** $k = 400$ recaptured $\alpha = 0.05$ (black), 0.02 (red), and 0.01 (blue). Notice that lower significance level α reduces power. **(right)** $\alpha = 0.05$, $k = 400$ (black), 600 (red), and 80000 (blue). Decreased significance level reduces power and increased recapture size increases power.

```
> N0<-2000
> ralpha<-qhyper(0.95,t,N0-t,k)
> data.frame(k,ralpha)
    k ralpha
1 400     49
2 600     70
3 800     91
```

We show the impact on power $\pi(N)$ of both significance level α and the number in the recapture k in Figure 2.

Exercise 4. *Determine the type II error rate for $N = 1600$ with*

- $k = 400$ *and* $\alpha = 0.05, 0.02,$ *and 0.01, and*
- $\alpha = 0.05$ *and* $k = 400, 600,$ *and 800.*

Example 5. *For a* **two-sided test**

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0.$$

In this case, the parameter values for the null hypothesis $\Theta_)$ consist of a single value, μ_0. We reject H_0 if $|\bar{X} - \mu_0|$ is too large. Again, with level α, we have the critical region

$$|Z| = \left|\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}\right| \geq z_{\alpha/2}.$$

If μ is the actual mean, then

$$\frac{\bar{X} - \mu}{\sigma_0/\sqrt{n}}.$$

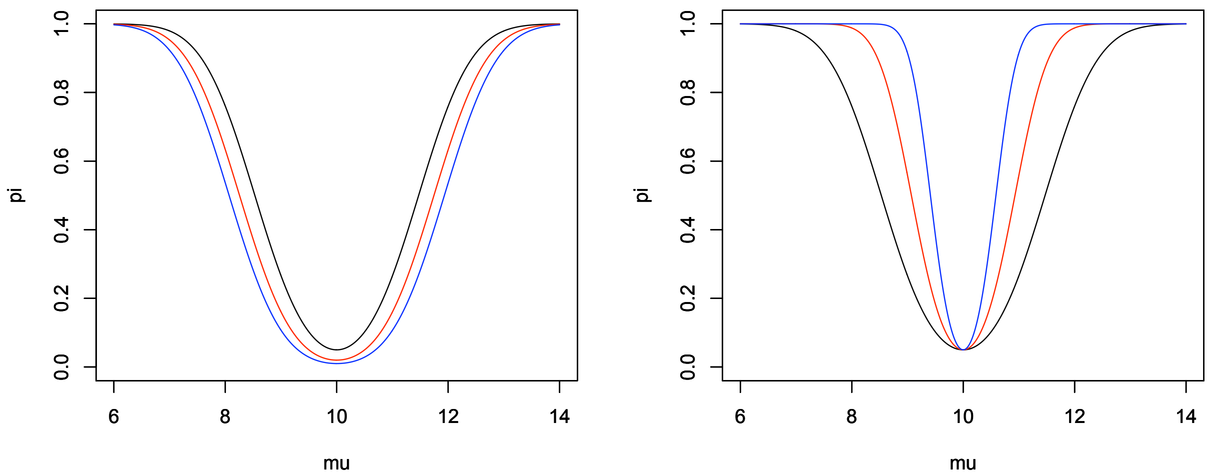

Figure 3: Power function for the two-sided test. $\mu_0 = 10$, $\sigma_0 = 3$. **(left)** $n = 16$, $\alpha = 0.05$ (black), 0.02 (red), and 0.01 (blue). Notice that lower significance level α reduces power. **(right)** $\alpha = 0.05$, $n = 15$ (black), 40 (red), and 100 (blue). As before, decreased significance level reduces power and increased sample size n increases power.

We use this fact to determine the power function for this test

$$\begin{aligned}
\pi(\mu) &= P_\mu\{X \in C\} = 1 - P_\mu\{X \notin C\} = 1 - P_\mu\left\{\left|\frac{\bar{X}-\mu_0}{\sigma_0/\sqrt{n}}\right| < z_{\alpha/2}\right\} \\
&= 1 - P_\mu\left\{-z_{\alpha/2} < \frac{\bar{X}-\mu_0}{\sigma_0/\sqrt{n}} < z_{\alpha/2}\right\} = 1 - P_\mu\left\{-z_{\alpha/2} - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}} < \frac{\bar{X}-\mu}{\sigma_0/\sqrt{n}} < z_{\alpha/2} - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}\right\} \\
&= 1 - \Phi\left(z_{\alpha/2} - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}\right) + \Phi\left(-z_{\alpha/2} - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}\right)
\end{aligned}$$

If we do not know if the mimic is larger or smaller that the model, then we use a two-sided test. Below is the R *commands for the power function with* $\alpha = 0.05$ *and* $n = 16$ *observations.*

```
> zalpha = qnorm(.975)
> mu0<-10
> sigma0<-3
> mu<-(600:1400)/100
> n<-16
> pi<-1-pnorm(zalpha-(mu-mu0)/(sigma0/sqrt(n)))+pnorm(-zalpha-(mu-mu0)/(sigma0/sqrt(n)))
> plot(mu,pi,type="l")
```

We shall see in the the next topic how these tests follow from extensions of the likelihood ratio test for simple hypotheses.

The next example is unlikely to occur in any genuine scientific situation. It is included because it allows us to compute the power function explicitly from the distribution of the test statistic. We begin with an exercise.

Exercise 6. *For* $X_1, X_2, \ldots, X_n$ *independent* $U(0,\theta)$ *random variables,* $\theta \in \Theta = (0,\infty)$. *The density*

$$f_X(x|\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x \le \theta, \\ 0 & \text{otherwise.} \end{cases}$$

Let $X_{(n)}$ denote the maximum of $X_1, X_2, \ldots, X_n$, then $X_{(n)}$ has distribution function

$$F_{X_{(n)}}(x) = P_\theta\{X_{(n)} \leq x\} = \left(\frac{x}{\theta}\right)^n.$$

Example 7. *For $X_1, X_2, \ldots, X_n$ independent $U(0, \theta)$ random variables, take the null hypothesis that θ lands in some normal range of values $[\theta_L, \theta_R]$. The alternative is that θ lies outside the normal range.*

$$H_0 : \theta_L \leq \theta \leq \theta_R \quad \text{versus} \quad H_1 : \theta < \theta_L \text{ or } \theta > \theta_R.$$

If any of our observations X_i are greater than θ_R, then we are certain $\theta > \theta_R$ and we should reject H_0. On the other hand, all of the observations could be below θ_L and the state of nature θ might still land in the normal range.

Consequently, we will try to base a test based on the statistic $X_{(n)} = \max_{1 \leq i \leq n} X_i$ and reject H_0 if $X_{(n)} > \theta_R$ and too much smaller than θ_L, say $\tilde{\theta}$. We shall soon see that the choice of $\tilde{\theta}$ will depend on n the number of observations and on α, the size of the test.

The power function

$$\pi(\theta) = P_\theta\{X_{(n)} \leq \tilde{\theta}\} + P_\theta\{X_{(n)} \geq \theta_R\}$$

We compute the power function in three cases. The second case has the values of θ under the null hypothesis. The first and the third cases have the values for θ under the alternative hypothesis. An example of the power function is shown in Figure 3.

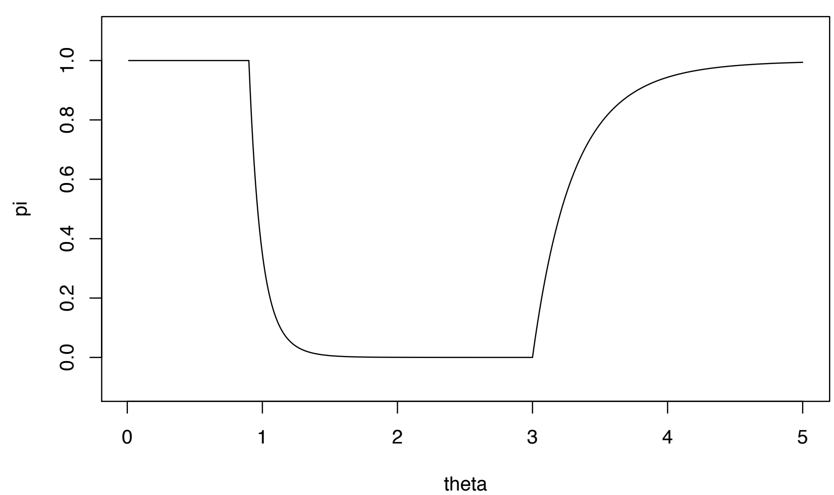

Figure 4: Power function for the test above with $\theta_L = 1, \theta_R = 3, \tilde{\theta} = 0.9$, and $n = 10$. The size of the test is $\pi(1) = 0.3487$.

Case 1. $\theta \leq \tilde{\theta}$.

In this case all of the observations X_i must be less than θ which is in turn less than $\tilde{\theta}$. Thus, $X_{(n)}$ is certainly less than $\tilde{\theta}$ and

$$P_\theta\{X_{(n)} \leq \tilde{\theta}\} = 1 \text{ and } P_\theta\{X_{(n)} \geq \theta_R\} = 0$$

and therefore $\pi(\theta) = 1$.

Case 2. $\tilde{\theta} < \theta \leq \theta_R$.

$$P_\theta\{X_{(n)} \le \tilde{\theta}\} = \left(\frac{\tilde{\theta}}{\theta}\right)^n \text{ and } P_\theta\{X_{(n)} \ge \theta_R\} = 0$$

and therefore $\pi(\theta) = (\tilde{\theta}/\theta)^n$.

Case 3. $\theta > \theta_R$.
Repeat the argument in Case 2 to conclude that

$$P_\theta\{X_{(n)} \le \tilde{\theta}\} = \left(\frac{\tilde{\theta}}{\theta}\right)^n$$

and that

$$P_\theta\{X_{(n)} \ge \theta_R\} = 1 - P_\theta\{X_{(n)} < \theta_R\} = 1 - \left(\frac{\theta_R}{\theta}\right)^n$$

and therefore $\pi(\theta) = (\tilde{\theta}/\theta)^n + 1 - (\theta_R/\theta)^n$.

The size of the test is the maximum value of the power function under the null hypothesis. This is case 2. Here, the power function

$$\pi(\theta) = \left(\frac{\tilde{\theta}}{\theta}\right)^n$$

decreases as a function of θ*. Thus, its maximum value takes place at* θ_L *and*

$$\alpha = \pi(\theta_L) = \left(\frac{\tilde{\theta}_L}{\theta}\right)^n$$

To achieve this level, we solve of $\tilde{\theta}$ *and take* $\tilde{\theta} = \theta_L \sqrt[n]{\alpha}$*. Note that* $\tilde{\theta}$ *increases with* α*. Consequently, we must reduce the critical region in order to reduce the significance level. Also,* $\tilde{\theta}$ *increases with* n *and we can reduce the critical region while maintaining significance if we increase the sample size.*

2 The p-value

The report of *reject* the null hypothesis does not describe the strength of the evidence because it fails to give us the sense of whether or not a small change in the values in the data could have resulted in a different decision. Consequently, one common method is not to choose, in advance, a significance level α of the test and then report "reject" or "fail to reject", but rather to report the value of the test statistic and to give all the values for α that would lead to the rejection of H_0. The p-value is the probability of obtaining a result at least as extreme as the one that was actually observed, assuming that the null hypothesis is true and measures the strength of evidence against H_0. Consequently, a very low p-value indicates strong evidence against the null hypothesis.

If the p-value is below a given significance level α, then we say that the result is **statistically significant** at the level α.

For example, if the test is based on having a test statistic $S(X)$ exceed a level k, i.e., we have decision

$$\text{reject } H_0 \text{ if and only if } S(X) \ge k.$$

and if the value $S(X) = k_0$ is observed, then the p-value equals to the lowest value of significance level that wold result in rejection of the null hypothesis.

$$\max\{\pi(\theta); \theta \in \Theta_0\} = \max\{P_\theta\{S(X) \ge k_0\}; \theta \in \Theta_0\}.$$

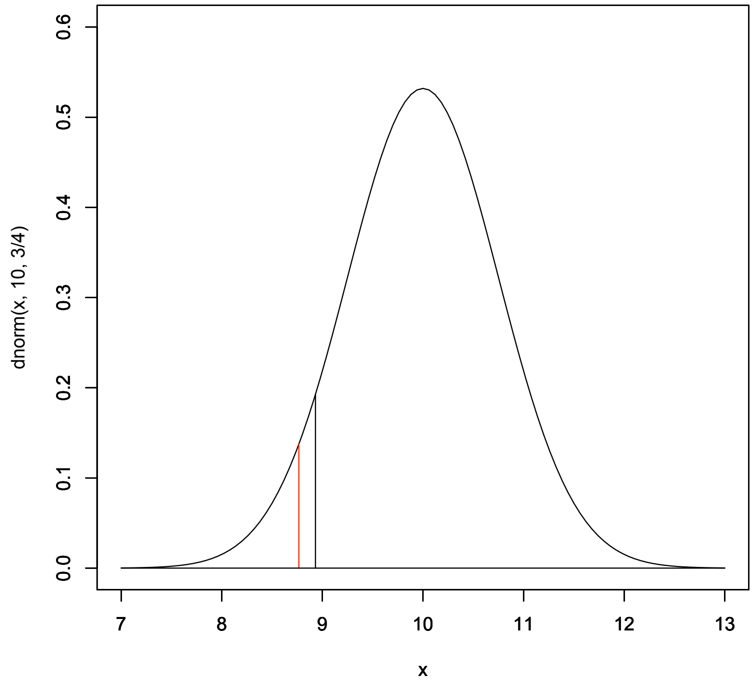

Figure 5: Under the null hypothesis, $\bar{X}$ has a normal distribution mean $\mu_0 = 10$cm, standard deviation $3/\sqrt{16} = 3/4$cm. The p-value, 0.077, is the area under the density curve to the left of the observed value of 8.931 for $\bar{x}$, The critical value, 8.767, for an $\alpha = 0.05$ level test is indicated by the red line. Because the p-vlaue is greater than the significance level, we cannot reject H_0.

For the one-sided hypothesis test to see if the mimic had invaded,

$$H_0 : \mu \geq \mu_0 \quad \text{versus} \quad H_1 : \mu < \mu_0.$$

with $\mu_0 = 10$ cm, $\sigma_0 = 3$ cm and $n = 16$ observations. Our data had sample mean $\bar{x} = 8.93125$ cm. The maximum value of the power function $\pi(\mu)$ for μ in the subset of the parameter space determined by the null hypothesis occurs for $\mu = \mu_0$. Consequently, the p-value is

$$P_{\mu_0}\{\bar{X} \leq 8.93125\}.$$

With the parameter value $\mu_0 = 10$ cm, $\bar{X}$ has mean 10 cm and standard deviation $3/\sqrt{16} = 3/4$. We can compute the p-value using R].

```
> pnorm(8.93125,10,3/4)
[1] 0.0770786
```

Thus, we cannot reject H_0 at the $\alpha = 0.05$ significance level. Indeed, we can reject H_0 at any level below the p-value.

Example 8. *Returning to the example on the proportion of hives that survive the winter, the appropriate composite hypothesis test to see if more that the usual normal of hives survive is*

$$H_0 : p \leq 0.7 \quad \textit{versus} \quad H_1 : p > 0.7.$$

The R *output shows a p-value of 3%.*

```
>  prop.test(88,112,0.7,alternative="greater")

1-sample proportions test with continuity correction
```

```
data:  88 out of 112, null probability 0.7
X-squared = 3.5208, df = 1, p-value = 0.0303
alternative hypothesis: true p is greater than 0.7
95 percent confidence interval:
 0.7107807 1.0000000
sample estimates:
        p
0.7857143
```

Exercise 9. *Is the hypothesis test above signfiicant at the 5% level? the 1% level?*

3 Answers to Selected Exercises

2. In this case the critical regions is $C = \{\mathbf{x}; \bar{x} \leq k(\mu_0)\}$ for some value $k(\mu_0)$. To find this value, note that

$$P_{\mu_0}\{Z \leq -z_\alpha\} = P_{\mu_0}\{\bar{X} \leq -\frac{\sigma_0}{\sqrt{n}} z_\alpha + \mu_0\}$$

and $k(\mu_0) = -(\sigma_0/\sqrt{n})z_\alpha + \mu_0$. The power function

$$\pi(\mu) = P_\mu\{\bar{X} \leq -\frac{\sigma_0}{\sqrt{n}} z_\alpha + \mu_0\} = P_\mu\{\bar{X} - \mu \leq -\frac{\sigma_0}{\sqrt{n}} z_\alpha - (\mu - \mu_0)\}$$
$$= P_\mu \left\{ \frac{\bar{X} - \mu}{\sigma_0/\sqrt{n}} \leq -z_\alpha - \frac{\mu - \mu_0}{\sigma_0/\sqrt{n}} \right\} = \Phi\left(-z_\alpha - \frac{\mu - \mu_0}{\sigma_0/\sqrt{n}}\right).$$

5. The type II error rate β is $1-\pi(1600) = P_{1600}\{R < r_\alpha\}$. This is the distribution function of a hypergeometric random variable and thus these probabilities can be computed using the `phyper` command

- For varying significance, we have the R commands:

```
> t<-200;N<-1600
> k<-400
> alpha<-c(0.05,0.02,0.01)
> ralpha<-c(49,51,53)
> beta<-phyper(ralpha,t,N-t,k)
> data.frame(alpha,beta)
  alpha      beta
1  0.05 0.4695591
2  0.02 0.6071656
3  0.01 0.7316226
```

  Notice that the type II error probability is high for $\alpha = 0.05$ and increases as α decreases.

- For varying recapture size, we continue with the R commands:

```
> k<-c(400,600,800)
> ralpha<-c(49,70,91)
> beta<-phyper(ralpha,t,N-t,k)
> data.frame(k,beta)
    k       beta
1 400 0.46955913
2 600 0.24194905
3 800 0.09933596
```

Notice that increasing recapture size has a significant impact on type II error probabilities.

6. The i-th observation satisfies

$$P\{X_i \le \tilde{\theta}\} = \int_0^{\tilde{\theta}} \frac{1}{\theta}\,dx = \frac{\tilde{\theta}}{\theta}$$

Now, $X_{(n)} \le \tilde{\theta}$ occurs precisely when all of the n-independent observations X_i satisfy $X_i \le \tilde{\theta}$. Because these random variables are independent,

$$\begin{aligned} F_{X_{(n)}} &= P_\theta\{X_{(n)} \le x\} = P_\theta\{X_1 \le x, X_1 \le x, \ldots, X_n \le x\} \\ &= P_\theta\{X_1 \le x\}P\{X_1 \le x\}, \cdots P\{X_n \le x\} = \left(\frac{x}{\theta}\right)\left(\frac{x}{\theta}\right)\cdots\left(\frac{x}{\theta}\right) = \left(\frac{x}{\theta}\right)^n \end{aligned}$$

9. Yes, the p-value is below 0.05. No, the p-value is above 0.01.