Extensions on the Likelihood Ratio Test[*]

Worksheet 21

1. Snell's law tell us how light bends at an interface - the angle of incidence versus the angle of refraction - based on the ratio of the velocities of light in the two isotropic media. If the angle of incidence of a laser beam in a vacuum is θ_1 radians and the angle of refraction in an unknown medium is θ_2, then

$$n = \frac{\sin \theta_1}{\sin \theta_2}$$

is called the **index of refraction** ($\beta = 1/n$ is the velocity of light in the medium as a fraction of the speed of light in a vacuum.) Make repeated independent measurements in radians, $\theta_{1,1}, \theta_{1,2}, \cdots, \theta_{1,16}$ of the angle of incidence in vacuum and $\theta_{2,1}, \theta_{2,2}, \cdots, \theta_{2,16}$, of the angle of refraction in the second medium. If these measurements have

- sample mean $\bar{\theta}_1 = 0.786$ radians and standard deviation 0.03 radians in vacuum and
- sample mean $\bar{\theta}_2 = 0.326$ radians and standard deviation 0.06 radians in the unknown medium.

(a) Snell's law gives an estimate $\hat{n}$ based on the values of $\bar{\theta}_1$ and $\bar{\theta}_2$. Use the delta method to estimate the mean and standard deviation of $\hat{n}$.

(b) You suspect that the substance is cubic zirconia ($n_z = 2.165$) and not the claimed material, diamond ($n_d = 2.418$). For the hypothesis

$$H_0 : n = n_d \quad \text{versus} \quad H_1 : n = n_z,$$

use the information above to devise a z-test for the hypothesis and report a p-value for the test.

(c) Describe what the p-value communicates in this case.

2. Migrants are thought to be more likely than native-born Americans to own a business,

(a) Give a hypothesis that is appropriate to this situation. Define any population parameters that you use in stating the hypothesis

A 2012 study based on Current Population Survey (CPS) data from the U.S. Census Bureau, found that the existing business-ownership rate is higher for migrants than native-born adults in America: 10.5% of the migrant workforce own a business, compared with 9.3% of the native-born labor force.

(b) Fill in the following table for the power for the given alternatives. Assume that 9.3% of the native-born labor force own a business,

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migrant proportion	significance level	power	number of observations
0.105	0.05	0.90	
0.105	0.01	0.95	
0.105	0.05		1000
0.105	0.05		2000
0.120	0.05	0.90	
0.120	0.01	0.95	
0.120	0.05		1000
0.120	0.05		2000