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- Example: There are three dogs, Rex, Bruno, and Fifi, and Bruno is the only black dog. If $P(x) =$ dog x is black, then $P(Rex) = F$, $P(Bruno) = T$ and $P(Fifi) = F$.

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- Quantifiers turn predicates into propositions:
  Existential ("there exists"): $\exists x P(x)$ is true if there is at least one x so $P(x)$ is true.
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- Let $P(x) =$ dog x is black. Which are true quantified statements?
  "There is a dog which is black." $\exists x P(x)$.
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- "There is a day of the week when I'm not working." Predicate is $W(x)$: I'm working on day x. In symbols $\exists x \neg W(x)$.

To check the truth of quantified statements:

- $\exists x P(x)$ is true if you can find one specific x so that $P(x) = T$.
- $\forall x P(x)$ is true if $P(x) = T$ for an *arbitrary* x.
- $\exists x P(x)$ is false if $P(x) = F$ for an *arbitrary* x.
- $\forall x P(x)$ is false $P(x) = F$ for some x ("counterexample").

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Identify the quantifier and show how to prove or disprove:

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- I can find a real number x so that $x^2 + 1 = 0$.

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Identify the quantifier and show how to prove or disprove:

- "There is a mammal that can fly."
- I can find a real number x so that $x^2 + 1 = 0$.
- "Every month has more than 28 days."
- Every even integer can be written as the sum of an odd integer plus one.

Suppose x is drawn from a set $x_1, x_2, \ldots x_n$. Then

$$\exists x P(x) = P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$$

and

$$\forall P(x) = P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n).$$

If n is small enough, can verify truth/falsehood by exhaustion of cases $P(x_1), P(x_2), \ldots$.

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- If a variable has no quantifier it is free, e.g. $P(x)$, otherwise it is bound as in $\exists x(A(x) \wedge B(x))$.
- Careful! variable must either be free or bound.
  Example: $B(x) \vee \exists x A(x)$ makes no sense.

Translate from English:
"All students who get an A on the final will pass the course."
"All students will get an A on the final or not pass the course."
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What does $\exists x(P(x) \wedge \neg A(x))$ say?
"There is a student who passed but did not get A on the final."

Check if the following statements are true. Give an example to show existential statement is true, or counterexample to show universal statement is false.

x	P	Q	R
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

(A) $\forall x (R(x) \vee Q(x)) \rightarrow P(x)$.

Chapter 1.7: Quantified statements, examples

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Recall

$$\exists x P(x) = P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n),$$
$$\forall x P(x) = P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n).$$

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Negations of these use extended form of DeMorgan's laws

$$\neg(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)) = \neg P(x_1) \wedge \neg P(x_2) \wedge \ldots \wedge \neg P(x_n),$$

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Therefore the negation of qualified statements is

$$\neg \exists x P(x) = \forall x \neg P(x), \quad \neg \forall x P(x) = \exists x \neg P(x).$$

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- Let $P(x)$: person x codes in python, $C(x)$: x codes in C++.
  Write the following in symbols, find the negation, and translate back to English:
  (a) "There is an employee that codes in python but not C++."
  (b) "Every employee that codes in python codes in C++".

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Chapter 1.9 Nested quantifiers

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  $\forall y \exists x P(x, y)$ means "Every day, someone misses work"
  $\exists x \forall y \neg P(x, y)$ means "There is a person who does not ever miss work."
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(A) Let $D(x, y)$: person x dislikes class y. Write in symbols:
"Each student has at least one class they dislike."
"There is a class that every student dislikes."

Chapter 1.9 Nested quantifiers, problems

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(B) Given the table of values for $P(x, y)$

P	a	b	c
d	T	F	T
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determine the truth for $\exists x \exists y P(x, y)$, $\forall x \exists y P(x, y)$, $\exists y \forall x P(x, y)$.

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(C) Write in symbols, and find the truth value:
"For any integer $x \geq 2$, there exists a prime number $y \leq x$ so that x is divisible by y."

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determine the truth for $\exists x \exists y P(x, y)$, $\forall x \exists y P(x, y)$, $\exists y \forall x P(x, y)$.

(C) Write in symbols, and find the truth value:
"For any integer $x \geq 2$, there exists a prime number $y \leq x$ so that x is divisible by y."

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  Ans: "Everyone is unhappy sometimes."

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- Need multiple quantifiers and DIFFERENT variable names, even if predicate takes a single input.

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(C) Consider a group of employees at a company. Let
$P(x, y)$: x knows y's phone number.
$D(x)$: x missed the deadline.
$N(x)$: x is a new employee.
"There is a new employee who knows everyone's phone number."
"Exactly one new employee missed the deadline."
"Except for two employees, if someone is new they missed the deadline."

- An argument is a sequence of propositions ("hypotheses") $p_1, p_2, \ldots$ and a concluding proposition q. For example: $p_1 =$ "Night is dark", $p_2 =$ "Night is cold", $q =$ "Night is dark and cold".

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Write as a symbolic argument, and check its validity.

(A) “If it is raining, I will take an umbrella. It is not raining. Therefore I did not take an umbrella.”

(B) ”It is not true that cats sleep when it is night. If cats sleep it is night. Therefore cats don’t sleep.”
(interpret “when” as biconditional)

(C) ”A number x is rational or irrational. x is an integer or irrational. Therefore x is either rational or an integer.”

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- Example: "It is raining. If it is raining I will take an umbrella. Therefore I will take an umbrella." (MP)

Rule of inference	Name
p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
p $\therefore p \lor q$	Addition
$p \land q$ $\therefore p$	Simplification
p q $\therefore p \land q$	Conjunction
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	Resolution

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Ex: "I will buy a car and a house only if I get a job. I am not going to get a job. I will buy a house. Therefore I will not buy a car."

$(C \wedge H) \rightarrow J$
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H (Hyp)

Ex: "I will buy a car and a house only if I get a job. I am not going to get a job. I will buy a house. Therefore I will not buy a car."

$(C \wedge H) \to J$

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$H \to \neg C$ (Cond)

H (Hyp)

$\neg C$ (MP)

Alternative: use disjunctive syllogism after the DeMorgan step.

(A)

$$\begin{array}{l} p \rightarrow q \\ \underline{\neg(q \wedge r)} \\ p \rightarrow \neg r \end{array}$$

(B) Write in symbols, and prove validity:
"Today it will rain or be cool. Either it will not rain or be windy. It is not cool, therefore it will be windy."

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$\underline{\neg(q \wedge r)}$

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$r \vee c$

$\neg r \vee w$

$\underline{\neg c}$

$\therefore w$

Inference with quantifiers

Need a way of moving between quantified statements and propositions about individual elements:

Rule of Inference	Name	Example
c is an element (arbitrary or particular) $\forall x P(x)$ $\therefore P(c)$	Universal instantiation	Izar is a student in the class. Every student in the class completed the assignment. Therefore, Izar completed the assignment.
c is an arbitrary element $P(c)$ $\therefore \forall x P(x)$	Universal generalization	Let c be an arbitrary integer. $c \leq c^2$ Therefore, every integer is less than or equal to its square.
$\exists x P(x)$ $\therefore$ (c is a particular element) $\wedge P(c)$	Existential instantiation*	There is an integer that is equal to its square. Therefore, $c^2 = c$, for some integer c.
c is an element (arbitrary or particular) $P(c)$ $\therefore \exists x P(x)$	Existential generalization	Sam is a particular student in the class. Sam completed the assignment. Therefore, there is a student in the class who completed the assignment.

Examples:

- UI: "Every day in Tucson is warm. So tomorrow will be warm." or "So a random day in February will be warm."

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- UG: "If x is an arbitrary student, they passed the test. Therefore all students passed the test."

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$\forall x Q(x)$ (UG)

Inference with quantifiers, examples

Show $\forall x(P(x) \wedge Q(x)) \therefore \forall x P(x) \wedge \forall x Q(x)$ is valid.
Proof:
$\forall x(P(x) \wedge Q(x))$ (Hypothesis)
$P(c) \wedge Q(c)$ for an arbitrary element c (UI)
$P(c)$ (Simplification)
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$\exists x(U(x) \wedge D(x))$ (EG).

(A) "Every student who has a permission slip can go on the field trip. Every student has a permission slip. Therefore every student can go on the field trip."

(B) "Every student on the honor roll received an A. No student who got a detention received an A. No student who got a detention is on the honor roll."
Use $H(x), A(x), D(x)$ for whether a student is on honor roll, received an A, and got detention.