

Chapter 5.1: Boolean algebra

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- These are perfectly analogous to logical operations:

Boolean \cdot	Logical \wedge
$0 \cdot 0 = 0$	$F \wedge F = F$
$0 \cdot 1 = 0$	$F \wedge T = F$
$1 \cdot 0 = 0$	$T \wedge F = F$
$1 \cdot 1 = 1$	$T \wedge T = T$

Boolean $+$	Logical \vee
$0 + 0 = 0$	$F \vee F = F$
$0 + 1 = 1$	$F \vee T = T$
$1 + 0 = 1$	$T \vee F = T$
$1 + 1 = 1$	$T \vee T = T$

Boolean complement	Logical \neg
$\bar{0} = 1$	$\neg F = T$
$\bar{1} = 0$	$\neg T = F$

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for example

$$\overline{1 + 1 \cdot (1 + 0)} = \overline{1 + 1 \cdot \bar{1}} = \overline{1 + 1 \cdot 0} = \overline{1 + 0} = \bar{1} = 0.$$

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Chapter 5.1: Law of Boolean algebra

Laws of Boolean algebra are essentially the same as logic, sets:

Idempotent laws:	$x + x = x$	$x \cdot x = x$
Associative laws:	$(x + y) + z = x + (y + z)$	$(xy)z = x(yz)$
Commutative laws:	$x + y = y + x$	$xy = yx$
Distributive laws:	$x + yz = (x + y)(x + z)$	$x(y + z) = xy + xz$
Identity laws:	$x + 0 = x$	$x \cdot 1 = x$
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Double complement law:	$\overline{\overline{x}} = x$	
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 $= x \cdot (y + \overline{y})$ (distributive)

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Problem: Show $(\overline{a}\overline{b} + c) \cdot (a + b) \cdot \overline{(\overline{b} + ac)} = \overline{a}bc$.

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- Important observation: each row in a truth table for $f(x, y, z, \dots)$ corresponds to a minterm $m(x, y, z, \dots)$ so that $m = 1$ only for that row of the table.
For example $(x, y, z) = (0, 1, 1)$ has $\bar{x}yz$ as the corresponding minterm.

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- Problem: for $f(x, y, z) = \overline{(x + \bar{z})} \cdot (y + z)$, find truth table, and write f as the sum of minterms.

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Note the terms don't have to be minterms.
- An expression which is the product of sums of literals is called **conjunctive normal form**. (CNF) Example:
 $(x + \bar{z})(y + \bar{z} + x)xz$, but not $(x + yz)(xyz + \bar{z})$.
- Can convert to DNF, generally by using DeMorgan and/or the distributive rule, and simplifying using compliment/domination/identity laws.

- An expression which is the sum of products of literals is called **disjunctive normal form**.(DNF) Example: $x\bar{y} + \bar{x}yz + \bar{z}$.
Note the terms don't have to be minterms.
- An expression which is the product of sums of literals is called **conjunctive normal form**. (CNF) Example:
 $(x + \bar{z})(y + \bar{z} + x)xz$, but not $(x + yz)(xyz + \bar{z})$.
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- Can convert to CNF by factoring/distributive, and simplifying

Chapter 5.3: Normal forms, examples

Examples of converting to DNF:

(1) $\overline{xy} + \overline{x + y}$

Chapter 5.3: Normal forms, examples

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Chapter 5.3: Normal forms, examples

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Chapter 5.3: Normal forms, examples

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Chapter 5.3: Normal forms, examples

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Chapter 5.3: Normal forms from tables

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Taking the complement and applying DeMorgan,

$$f(x, y, z) = (x + y + z)(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + \bar{z})$$

(1) Find the DNF and CNF for the expression $\overline{(x + \bar{y})}y$.

(2) Suppose that a boolean function $f(x, y, z, w)$ is equal to one, except when $(x, y, z, w) = (1, 0, 1, 0)$ or $(x, y, z, w) = (0, 1, 0, 1)$. Determine a CNF expression for f .

- Consider a (compound) expression in propositional logic, depending on variables x, y, z, \dots . Recall that if it is always false, this expression is a contradiction. This means the corresponding Boolean function would always be 0.

Chapter 5.5: Boolean satisfiability

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Chapter 5.5: Boolean satisfiability, a simple problem

Suppose there are three switches (up=1,down=0), and a light turns on provided:

- (1) if switch 1 is down, then switch 2 must be up;
- (2) both switches 2 and 3 must be in the same position.

The question is: in what circumstances is the light on?

Let x, y, z represent the state of switches 1,2,3, resp. In terms of logic, let X, Y, Z be the propositions that x, y, z are in the up position. Then we want

$$(\neg X \rightarrow Y) \wedge ((Y \wedge Z) \vee (\neg Y \wedge \neg Z)) = T.$$

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$$f(x, y, z) = (x + y)(yz + \bar{y}\bar{z}) = 1.$$

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Problem: convert this to DNF. Answer $f = xyz + x\bar{y}\bar{z} + yz$. There are four cases where $f = 1$: $(x, y, z) = (1, 1, 1), (1, 0, 0)$ and $(x, 1, 1)$ where x is arbitrary.

Suppose there are four workers in a factory, who must be assigned to machines A or B . The constraints are:

- (1) Each machine must have one worker.
- (2) Workers 1 and 3 cant be on the same machine.
- (3) Workers 2 and 4 must be on the same machine.
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Let x_1, x_2, x_3, x_4 be Boolean variables, where $x_k = 1$ when worker k is assigned to machine B . Machines A and B must each have a worker, so $x_1 + x_2 + x_3 + x_4$ and $\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4$ must be one. The other constraints imply $x_1\bar{x}_3 + x_3\bar{x}_1$, $x_2x_4 + \bar{x}_2\bar{x}_4$ and $x_1 + x_2$ must be equal to one.

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Putting this together, the question becomes one of satisfiability of

$$f = (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)(x_1 + x_2 + x_3 + x_4)(x_1\bar{x}_3 + x_3\bar{x}_1)(x_2x_4 + \bar{x}_2\bar{x}_4)(x_1 + x_2)$$

CNF using my computer (!):

YOUR INPUT

Simplify the boolean expression $(\overline{X_1} + \overline{X_2} + \overline{X_3} + \overline{X_4}) \cdot (X_1 + X_2 + X_3 + X_4) \cdot ((X_1 \cdot \overline{X_3}) + (X_3 \cdot \overline{X_1})) \cdot ((X_2 \cdot X_4) + (\overline{X_2} \cdot \overline{X_4})) \cdot (X_1 + X_2)$.

SOLUTION

The solution is too long.

ANSWER

$$(\overline{X_1} + \overline{X_2} + \overline{X_3} + \overline{X_4}) \cdot (X_1 + X_2 + X_3 + X_4) \cdot ((X_1 \cdot \overline{X_3}) + (X_3 \cdot \overline{X_1})) \cdot ((X_2 \cdot X_4) + (\overline{X_2} \cdot \overline{X_4})) \cdot (X_1 + X_2) = (X_1 \cdot X_4 \cdot X_2 \cdot \overline{X_3}) + (X_3 \cdot X_4 \cdot X_2 \cdot \overline{X_1}) + (X_1 \cdot \overline{X_2} \cdot \overline{X_3} \cdot \overline{X_4})$$

The DNF is $(X_1 \cdot X_2 \cdot X_4 \cdot \overline{X_3}) + (X_2 \cdot X_3 \cdot X_4 \cdot \overline{X_1}) + (X_1 \cdot \overline{X_2} \cdot \overline{X_3} \cdot \overline{X_4})$.

The CNF is $(X_1 + X_2) \cdot (X_1 + X_3) \cdot (X_2 + \overline{X_4}) \cdot (X_4 + \overline{X_2}) \cdot (\overline{X_1} + \overline{X_3})$.

Therefore the allowed assignments are:

A- worker 3, B - workers 1,2,4

A - worker 1, B - workers 2,3,4

A - workers 2,3,4, B - worker 1