Math 129 Section 005H
Lecture 3: Integration by parts (continued)

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Note: Here is a PDF version of this file.
This note follows and supplements Section 7.2 of the text.

Admin

- Office hour today at 1-2pm on Zoom.
- Math Department tutoring (online for the semester) starts next week
- Quiz will be on Gradescope on Tuesday

Last time: integration by parts

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x). \tag{1}$$

Definite integrals:

$$\int_a^b u(x)v'(x)\,dx = u(b)v(b) - u(a)v(a) - \int_a^b u'(x)v(x). \tag{2}$$

Tricky examples

1)

$$\int x^3 e^{x^2}\ dx$$

(This is from last time.) There are two choices: substituion then integrate by parts, or integrate by parts then sub.

2)

$$\int e^x \cos(x)\ dx$$

Note: this is a standard example where we integrate by parts twice and do a bit of algebra to find an answer.

3)

$$\int \ln(x)\ dx$$

Key is to see this as $\int \ln(x)\ dx = \int 1 \cdot \ln(x)\ dx$...

Your turn

1)

$$\int x^5 \sin(x^3)\ dx$$

2)

$$\int_0^{2\pi} \sin^2(x)\ dx$$

In class we did this without the limits. It may look like one where you integrate by parts twice. However, if you do that, you get $0 = 0$! The trig identity $\cos^2(x) + \sin^2(x) = 1$ is very useful in this problem.

3)

$$\int \arctan(x)\ dx$$

Again, key is to see this as $\int 1 \cdot \arctan(x)\ dx$.

Tabular (optional)

Tabular integration is useful for repeated integration by parts where $u(x)$ is a polynomial. For example:

$$\int x^5 e^{2x}\,dx$$

can be done by putting all the u's in one column and all the v''s in another, like this:

	u	v'
		e^{2x}
+	x^5	$\frac{1}{2}e^{2x}$
-	$5x^4$	$\frac{1}{4}e^{2x}$
+	$20x^3$	$\frac{1}{8}e^{2x}$
-	$60x^2$	$\frac{1}{16}e^{2x}$
+	$120x$	$\frac{1}{32}e^{2x}$
-	120	$\frac{1}{64}e^{2x}$

In the first column we put alternating ± signs. Notice how we line up the two columns. Then the antiderivative is

$$\begin{aligned} & x^5 \cdot \tfrac{1}{2}e^{2x} \\ - & \; 5x^4 \cdot \tfrac{1}{4}e^{2x} \\ + & \; 20x^3 \cdot \tfrac{1}{8}e^{2x} \\ - & \; 60x^2 \cdot \tfrac{1}{16}e^{2x} \\ + & \; 120x \cdot \tfrac{1}{32}e^{2x} \\ - & \; 120 \cdot \tfrac{1}{64}e^{2x} + C \end{aligned}$$

$$= \left(\frac{1}{2}x^5 - \frac{5}{4}x^4 + \frac{5}{2}x^3 - \frac{15}{4}x^2 + \frac{15}{4}x - \frac{15}{8}\right)e^{2x} + C.$$