YOUR NAME: SOLUTION

Math 129-005H Quiz 7[1]

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Here is a PDF version of this document.

Instructions. You all know the drill by now. Here is the Gradescope link. **Quizzes are closed book and closed notes. You may consult the table of integrals in your text. Otherwise, all normal exam rules apply. You must show all key steps to receive full credit.**

You may also use the following facts:

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \; ; \qquad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \; ; \qquad \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \; ;$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \cdots \, , \quad p \neq \text{positive integer.}$$

1) Find the first three nonzero terms of the Taylor series of

$$\sin(x) \cdot \frac{1}{1-x}$$

about $x = 0$.

Solution: since

$$\begin{aligned} \sin(x) &= x - \frac{x^3}{3!} + \cdots \\ \frac{1}{1-x} &= 1 + x + x^2 + \cdots \\ \frac{\sin(x)}{1-x} &= \left(x - \frac{x^3}{3!} + \cdots\right) \cdot \left(1 + x + x^2 + \cdots\right) \end{aligned}$$

Cross multiply, starting with the lowest degree terms:

$$\begin{array}{cl} x \cdot 1 & \text{(only one way to have an } x \text{ term)} \\ +x \cdot x & \text{(only one way to have an } x^2 \text{ term)} \\ +x \cdot x^2 - \frac{x^3}{6} \cdot 1 & \text{(two ways to have } x^3 \text{ terms)} \\ + \cdots & \end{array}$$

so $\boxed{x + x^2 + \frac{5}{6} x^3 + \cdots}$.

(more on next page)

2) Find $P_3(x)$ about $x = 0$ for

$$\sqrt[3]{\cos(x)}.$$

Solution: as in the written HW, use

$$\begin{aligned} (1+y)^p &= 1 + py + \frac{p(p-1)}{2!}y^2 + \frac{p(p-1)(p-2)}{3!}y^3 + \cdots \\ & \quad \text{with } p = 1/3 \\ \cos(x) &= 1 - \frac{x^2}{2!} + \cdots \end{aligned}$$

Let $y = \cos(x) - 1 = -\frac{x^2}{2} + \cdots$:

$$\begin{aligned} (1+y)^{1/3} &= 1 + \frac{1}{3}\Big(-\frac{x^2}{2} + \cdots\Big) + \frac{1}{3}\cdot\Big(-\frac{2}{3}\Big)\Big(-\frac{x^2}{2} + \cdots\Big)^2 + \cdots \\ &= \boxed{1 - \tfrac{1}{6}x^2 + \cdots} \end{aligned}$$

(There are no cubic terms, so this is it for $P_3(x)$.)