teach-322

Syllabus: From linear algebra to PDEs

Objective

This course is designed to prepare students for studying the linear systems that arise in engineering, in particular linear time invariant systems, linear filtering in signal processing, etc. Though not a course in linear algebra, the course emphasizes general principles of linearity and superposition as well as orthogonality.

Students should have had MATH 254 coming in. This course is a prereq for OPTI 330.

Students who successfully complete the course should

1. know the different ways of determining when a system of linear algebraic equations has a solution, and whether that solution is unique;
2. know the geometric interpretation of the solution sets of linear algebraic equations;
3. be able to solve systems of linear algebraic equations by gaussian elimination;
4. be able to solve 2x2 and 3x3 homogeneous linear systems of ODEs by diagonalizing the matrix;
5. be able to solve initial value problems for nonhomogeneous linear systems of ODEs breaking the solution into homogeneous and particular solutions;
6. be able to find particular solutions of nonhomogeneous equations using appropriate methods;
7. know how to compute Fourier series expansions for general periodic functions;
8. be able to solve basic linear PDEs like heat and wave equations on the interval (with Dirichlet or Neumann boundary conditions) using Fourier series; and
9. be comfortable computing the Fourier transforms of "signals" (time dependent functions) both by direct computation and by using properties of Fourier transform, in particular the convolution theorem.

They should appreicate superposition and orthogonality as useful general principles when dealing with linear systems, even if they cannot necessarily articulate these concepts in precise mathematical terms.

What this course is NOT:

1. It's not a course in linear algebra. In fact, it's important to limit how much time one spends on linear algebra, otherwise one could not get through the later material.
2. It's not a course in linear PDEs. In particular, I cut out Sturm-Liouville equations to ensure there is enough time for Fourier transforms.

Week 1

13.1 Complex arithmetic

13.2 Polar coordinates; 13.5 Complex exponential

13.5 Complex exponential

We need facility with complex numbers so that we can talk about complex eigenvalues later. We also need the complex exponential for solving linear ODEs. Trig functions and logarithm are just here to given them a bit of extra practice with complex numbers.

Week 2

~~13.6 Trig functions; 13.7 Logarithms~~

7.3 Linear systems; 7.1 Matrices, vectors

Kreyszig jumps right into matrix and vector algebra, which is a little abstract since students are not expected to have had linear algebra. I suggest instead to start with the first part of 7.3, using linear systems of equations to motivate matrices. Then cover vector and matrix algebra. After that, one can return to cover the rest of 7.3, i.e., gaussian elimination.

7.2 Matrix multiplication

Again, one can use linear systems of equations (i.e., substitutions) to motivate matrix multiplication.

7.2 More on matrix products

Lots of examples

Week 3

7.3 Gaussian elimination

Students need this later to find eigenvectors, so it's important they know what to do with underdetermined systems.

7.3 More examples of Gaussian elimination + geometric interpretation of solutions

7.4 Linear independence

Week 4

7.5 Existence & uniqueness of solutions

7.8 Inverse matrices

7.6 Determinants in the 2x2 and 3x3 cases

One can cover 7.7 if there is time, but most of the rest of the course will end up dealing with 3x3 systems, I think.

Week 5

~~7.7 general determinants~~

Review: Inverse matrix theorem

It's useful to summarize all the equivalent conditions for a system of linear algebraic equations to have 0, 1, or infinitely solutions. This is not done explicitly in the text.

Exam 1

7.9 Vector spaces, inner products

The axioms of vector spaces is not essential to what follows, but it doesn't hurt to define vector spaces either. The notion of inner product spaces and orthogonality will be useful when discussing Fourier series.

Week 6

7.9 More on vector spaces and inner products

8.1 Eigenvalue problem

This will be used to solve systems of ODEs, so students need to know how to deal with complex eigenvalues and eigenvectors.

8.1 More on eigenvalues and eigenvecors

Week 7

8.2 Applications of eigenvalue problems

2.2 Const-coeff homogeneous linear 2nd-order ODEs

This should be a review for most students. In this section and the next, I would suggest emphasizing superposition as a general principle, and breaking solutions down to homogeneous and particular solutions. Even without having formally had linear algebra, I think it would make sense to the students.

2.7 Nonhomogeneous ODEs

Week 8

2.7 More examples; 3.2 Higher-order homogeneous linear ODEs

3.2 continued

~~3.3 nonhomogeneous linear ODEs~~

4.1 Linear systems of ODEs

This is one of the key parts of the course.

Week 9

4.1 continued; 4.2 Basic theory

4.2 continued; 4.3 Const-ceff systems

4.3 continued

Week 10

4.6 Nonhomogeneous systems

When to use which method can be hard for students.

Exam 2

11.1 Fourier series

Though the book no longer covers complex Fourier series (it existed in editions prior to 10), many of the instructors do cover it. By this point, the students should have the background needed to understand it.

Week 11

11.2 More Fourier series

11.3 Forced oscillations

11.4 Trig polynomial approximations

Week 12

11.7 Fourier integral

11.9 Fourier transform

DFT and FFT can be skipped. Convolution theorem and other properties of the Fourier transform, on the other hand, are very important for many of the engineering students.

11.9 More on transforms

Week 13

11.10 Tables of transforms

Exam 3

12.1 Basic concepts of PDEs; 12.2 Modeling

Week 14

12.2 continued

12.3 Separation of variables, Fourier series

12.3 continued

Week 15

12.5, 12.6 Heat equation

Review

Author: K K Lin
Created: 2017-08-03 Thu 20:41
Emacs 25.2.1 (Org mode 8.2.10)
Validate