Homework 4 additional problem

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Consider the Markov chain with state space $S = \{0, 1, 2, \cdots\}$ and transition probabilities

$$p(i,j) = \begin{cases} p, & j = i+1 \\ q, & j = 0 \\ 0, & \text{otherwise} \end{cases}$$

where $p, q > 0$ and $p + q = 1$.[1] This example was discussed in class a few lectures ago; it counts the lengths of runs of heads in a sequence of independent coin tosses.

1) Show that the chain is irreducible.[2]

2) Find $P_0(T_0 = n)$ for $n = 1, 2, \cdots$.[3] What is the name of this distribution?

3) Is the chain recurrent? Explain.

[1] Be careful: this looks superficially similar to one of the gambling examples from the first week, but the state 0 is *not* absorbing.

[2] To show this, you need to show that for any pair of states x and y, (i) there is indeed a path from x to y (and describe that path); and (ii) show that there is a positive probability of the Markov chain taking that path.

[3] Here, by $P_0(T_0 = n)$ I mean $P(T_0 = n \mid X_0 = 0)$. In particular, $P_0(\cdot)$ does *not* refer to the initial distribution.