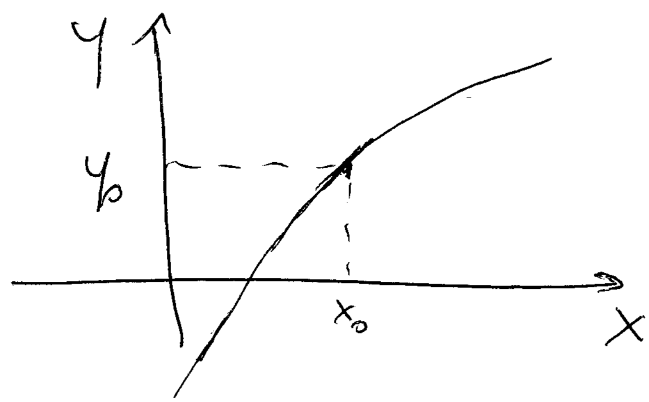
1. What does uniqueness mean?

If we have say a family of solution curves & if we pick a point (x_0, y_0) in the plane, then if there is a solution that goes through this point, there is only one.

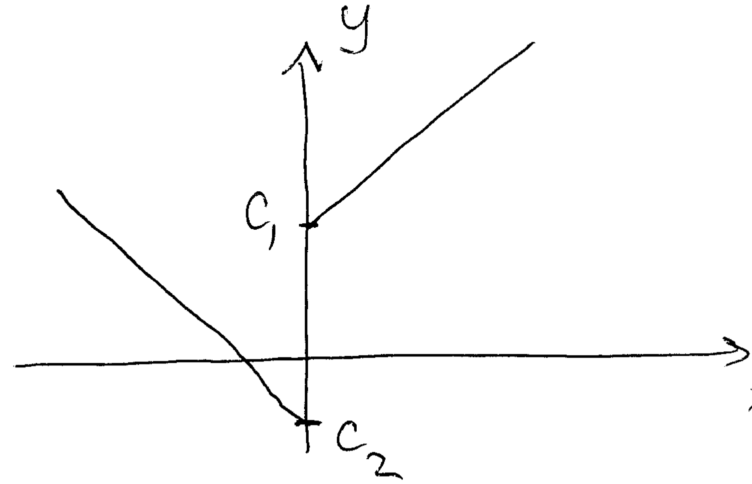


2. What can we do if g becomes singular?

Ex: Consider $y' = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases}$

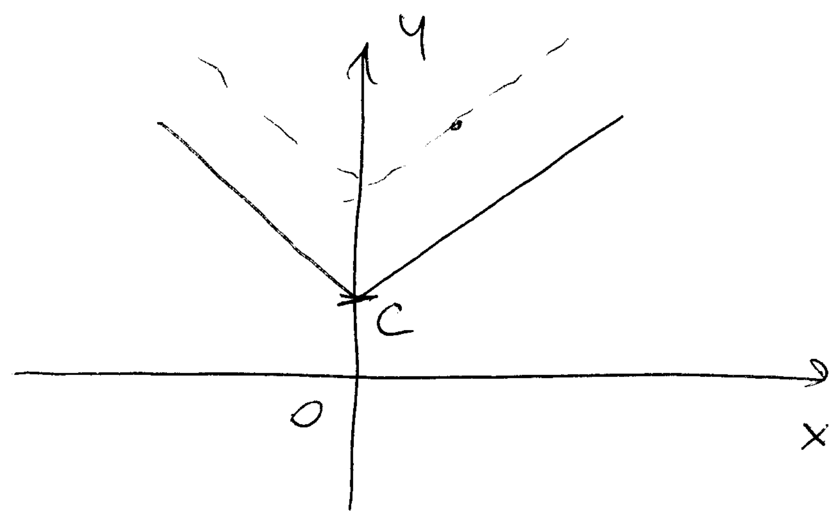
For $x \geq 0$ $y' = 1$ so $y(x) = x + C_1$

For $x < 0$ $y' = -1$ so $y(x) = -x + C_2$



If we want y to be continuous at $x=0$, we can <u>choose</u> $C_1 = C_2 = C$.

Then $y(x) = \begin{cases} x + C & \text{for } x \geq 0 \\ -x + C & \text{for } x < 0 \end{cases}$



3. $y' = g(x)$: <u>Qualitative properties</u>

We know when y increases or decreases. If g'' exists, we also know the concavity of solution curves.

<u>Symmetries</u>:

Assume that $g(x)$ is odd, i.e.

$$g(-x) = -g(x).$$

Let us assume that $y(x)$ solves

$$y' = g(x).$$

Let us define $u(x) = y(-x)$.

$$\frac{du}{dx} = -y'(-x) \underset{\text{use ode}}{=} -g(-x) \overset{g \text{ is odd}}{=} g(x)$$

So $u' = g(x)$, i.e. u solves the differential equation.