

## Methods of integration II (continued)

### 2. Trigonometric substitutions (continued)

Integrand of the form  $\sqrt{x^2 - a^2}$

$$\text{Set } x = a \cosh(\theta)$$

$$x^2 - a^2 = a^2 \sinh^2(\theta)$$

$$\begin{aligned} \text{Then, } \sqrt{x^2 - a^2} &= a \sqrt{\sinh^2(\theta)} = a |\sinh(\theta)| \\ &= a \sinh(\theta) \quad \text{since } \theta \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Therefore } \int \sqrt{x^2 - a^2} dx & \quad dx = a \sinh(\theta) d\theta \\ &= \int a \sinh(\theta) a \sinh(\theta) d\theta \\ &= a^2 \int \sinh^2(\theta) d\theta \end{aligned}$$

### Half-angle substitutions

$$\begin{aligned} \cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\ \sin(a+b) &= \cos(a)\sin(b) + \sin(a)\cos(b) \end{aligned}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2 \cos(x) \sin(x)$$

$$\begin{aligned} \cos(\theta) &= \frac{\cos(\theta)}{1} = \frac{\cos\left(2 \frac{\theta}{2}\right)}{1} \\ &= \frac{\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)} \\ &= \frac{1 - \frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}{1 + \frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}} \end{aligned}$$

Let  $t = \tan\left(\frac{\theta}{2}\right)$ . Then,  $\cos(\theta) = \frac{1-t^2}{1+t^2}$ .