Methods of integration II (continued)

2. Trigonometric substitutions (continued)

Integrand of the form $\sqrt{x^2-a^2}$

Set $x = a\cosh(\theta)$

$$x^2-a^2 = a^2\sinh^2(\theta)$$

Then, $\sqrt{x^2-a^2} = a\sqrt{\sinh^2(\theta)} = a\,|\sinh(\theta)|$

$$= a\sinh(\theta) \qquad \text{since } \theta \geq 0$$

Therefore $\int \sqrt{x^2-a^2}\,dx$ $\qquad dx = a\sinh(\theta)\,d\theta$

$$= \int a\sinh(\theta)\; a\sinh(\theta)\,d\theta$$

$$= a^2\int \sinh^2(\theta)\,d\theta$$

Half-angle substitutions

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a+b) = \cos(a)\sin(b) + \sin(a)\cos(b)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin(2x) = 2\cos(x)\sin(x)$$

$$\cos(\theta) = \frac{\cos(\theta)}{1} = \frac{\cos\left(2\,\frac{\theta}{2}\right)}{1}$$
$$= \frac{\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)}$$
$$= \frac{1 - \frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}{1 + \frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}$$

Let $t = \tan\left(\frac{\theta}{2}\right)$. Then, $\cos(\theta) = \dfrac{1-t^2}{1+t^2}$.