11/14/08

Numerical approximations (continued)

Midpoint rule $\displaystyle\int_{x_i}^{x_{i+1}} f(x)\,dx$

Let $m = \dfrac{x_i + x_{i+1}}{2}$

Write $f(x) = f(m) + (x-m)\,f'(m) + \dfrac{(x-m)^2}{2!} f''(m)$

$+ \dfrac{(x-m)^3}{3!} f'''(m) + \dfrac{(x-m)^4}{4!} f^{(4)}(\xi)$

ξ "between" x & m.

$$\int_{x_i}^{x_{i+1}} f(m)\,dx = f(m)\left(x_{i+1} - x_i\right) = f(m)\,\Delta x$$

$$\int_{x_i}^{x_{i+1}} (x-m) f'(m)\,dx = \left[f'(m) \frac{(x-m)^2}{2} \right]_{x_i}^{x_{i+1}}$$

$$= \frac{f'(m)}{2} \left[\frac{(x_{i+1}-m)^2}{2} - \frac{(x_i - m)^2}{2} \right] = 0$$