

Calculus and Differential Equations I

MATH 250 A

Maxima, minima, inflection points, and differentiability

Sketching the graph of a function

$$f(x) = 2x^3 - 3x^2 - 12x + 2$$

- Do you know how to **sketch the graph of this function** without using a calculator?
 - 1 Yes
 - 2 No
- If one **adds a constant** to the right hand side of the above equation, what does this do to the graph of f ?
 - 1 Translate the graph horizontally
 - 2 Translate the graph vertically
- What do you need to do to **translate the graph horizontally**?

Continuity and differentiability

- Can a function f be **continuous but not differentiable** at a point?
 - 1 Yes
 - 2 No
- Can a function f be **differentiable but not continuous** at a point?
 - 1 Yes
 - 2 No
- If f is a smooth function, **is it possible to have f' positive and f locally decreasing**?
 - 1 Yes
 - 2 No
- If f is a smooth function of x and $f''(x) > 0$, then
 - 1 f is concave up
 - 2 f is concave down
 - 3 One cannot tell

Local and global extrema

- A function f has a **local maximum** at a point x_0 if $f(x) \leq f(x_0)$ for x near x_0 .
- Similarly, a function f has a **local minimum** at a point x_0 if $f(x) \geq f(x_0)$ for x near x_0 .
- An extremum (minimum or maximum) of a function can be **local** or **global**.
- Does a function **always have** a global maximum?
 - 1 Yes
 - 2 No
- Is it possible for a function to have **more than one global minimum**?
 - 1 Yes
 - 2 No

Extrema and first derivative

- If a function f has a **local minimum at $x = 0$** , does it necessarily mean that $f'(0) = 0$?
 - 1 Yes
 - 2 No
- A function f has a **critical point** at $x = x_0$ if either $f'(x_0) = 0$ or $f'(x_0)$ is undefined.
- **Fact:** If f has a local maximum or minimum at $x = x_0$, if x_0 is not an end-point of the domain of definition of f , and if f is differentiable at $x = x_0$, then $f'(x_0) = 0$.
- We now consider the **converse of this statement**. What should we require of f' for f to have a local maximum or a local minimum at $x = x_0$?

Extrema and first derivative (continued)

- If f is differentiable and $f'(x_0) = 0$, does it mean that f has a local maximum or minimum at $x = x_0$?
 - 1 Yes
 - 2 No
- **First-derivative test:** if f has a critical point at $x = x_0$, is continuous at $x = x_0$, and is differentiable on the right and on the left of x_0 , then
 - If f' goes from negative to positive as x is increased past x_0 , then f has a local minimum at $x = x_0$.
 - If f' goes from positive to negative as x is increased past x_0 , then f has a local maximum at $x = x_0$.
- In particular, if f is twice differentiable on some interval, if f'' has a constant sign near x_0 , and if x_0 is not an end-point of the interval, then f has a local maximum or minimum at $x = x_0$ if and only if $f'(x_0) = 0$.

Extrema and second derivative, inflection points

- **Second-derivative test:** If f is twice-differentiable, if f'' is continuous, and if $f'(x_0) = 0$, then
 - If $f''(x_0) > 0$, then f has a local minimum at x_0 .
 - If $f''(x_0) < 0$, then f has a local maximum at x_0 .
 - If $f''(x_0) = 0$, then one cannot conclude the existence of a local extremum at $x = x_0$.
- An **inflection point** of a function f is a point x_0 such that **the graph of f changes concavity** at $x = x_0$.
- In other words, the graph of the function f **crosses its tangent** at an inflection point.
- How are inflection points and second derivatives related?

Inflection points and second derivative

- If f is twice-differentiable and x_0 is not an end-point of the domain of definition of f , **is the following statement true or false?**
Statement: If $f''(x_0) = 0$, then f has an inflection point at $x = x_0$.
 - 1 True
 - 2 False
- If f is twice-differentiable and if f'' changes sign when x goes through x_0 , then **f has an inflection point at $x = x_0$** .
- **Reading assignment:** Section 4.1 of the *Calculus* book.
- **Homework:** *Review of differentiation* in WebAssign.