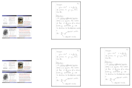
Calculus and Differential Equations I

MATH 250 A

Summary

What differential equations are and how we study them



- Examples of differential equations and of systems of differential equations.
  - Identify the independent and dependent variables, as well as the parameters, if any.
  - Characteristic properties: order, linear vs. nonlinear, autonomous vs. non-autonomous.
  - Initial or boundary conditions are often given.
- Questions to be addressed in the study of a differential equation (or a system of differential equations)
  - Existence and uniqueness
  - Geometric considerations
  - Numerical solutions
  - Analytical solutions

Existence and uniqueness

- Question: Given an initial condition, decide whether there exists a solution near the initial condition, and if so, whether it is unique.

- We have seen a variety of theorems that guarantee existence and/or uniqueness of solutions to differential equations.
  - For equations of the form $y' = g(x)$
  - For equations of the form $y' = g(y)$
  - For equations of the form $y' = g(x, y)$
  - For first order linear equations

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Geometric considerations

- Decide where solution curves increase, decrease, or are concave up or down
  - For equations of the form $y' = g(x)$
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  - You should be able to generalize the above to equations of the form $y' = g(x, y)$

- Symmetries of the family of solution curves $\mathcal{S}$ given symmetries of the differential equation $\mathcal{E}$
  - If $\mathcal{E}$ is invariant under $x \to -x$, then $\mathcal{S}$ is symmetric with respect to the y-axis.
  - If $\mathcal{E}$ is invariant under $y \to -y$, then $\mathcal{S}$ is symmetric with respect to the x-axis.
  - If $\mathcal{E}$ is invariant under $x \to -x$ and $y \to -y$, then $\mathcal{S}$ is symmetric with respect to the origin.

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Numerical solutions

- For equations of the form $y' = g(x)$, we have seen various ways of approximating integrals.
- For equations of the form $y' = g(x, y)$, we have discussed Euler's method.
- In both cases, we can sometimes decide whether an approximation is an underestimate or overestimate.
- We also discussed the various approximation errors associated with these methods.
- As part of the above, we introduced Taylor polynomials as ways to approximate functions and discussed the resulting error.

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Analytical solutions

- For equations of the form $y' = g(x)$
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- For equations of the form $y' = g(x, y)$
  - Separable equations
  - Equations with homogeneous coefficients
  - Linear equations
  - Bernoulli equations
- Solving a differential equation always involves evaluating an integral. We have seen various methods of integration
  - Substitutions
  - Integration by parts
  - Method of partial fractions
  - Trigonometric substitutions

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