

## Improper integrals (continued)

Example 3 :  $\int_1^{\infty} \frac{x^2+4}{x^4+3x^2+11} dx$

Since  $\frac{x^2+4}{x^4+3x^2+11}$  goes like  $\frac{1}{x^2}$  as  $x \rightarrow \infty$ ,

we expect the integral to converge.

So we want to find a function  $g(x)$

such that

$$0 \leq \frac{x^2+4}{x^4+3x^2+11} \leq g(x) \quad \text{and} \quad \int_1^{\infty} g(x) dx \text{ converges.}$$

Say  $\frac{x^2+4}{x^4+3x^2+11} \leq \frac{\alpha}{x^2} \quad \alpha > 0$

i.e.  $x^4+4x^2 \leq \alpha x^4 + 3\alpha x^2 + 11\alpha$

$$\Leftrightarrow 0 \leq (\alpha-1)x^4 + (3\alpha-4)x^2 + 11\alpha$$

Choose  $\alpha$  such that  $\alpha-1 > 0$  and  $3\alpha-4 > 0$

So for instance, choose  $\alpha = 2$ .