Improper integrals (continued)

$$\frac{x^2+4}{x^4+3x^2+11} = \frac{1}{x^2}\frac{x^4+4x^2}{x^4+3x^2+11}$$

$$= \frac{1}{x^2}\frac{(x^4+3x^2+11)+x^2-11}{x^4+3x^2+11} = \frac{1}{x^2}\left(1+\frac{x^2-11}{x^4+3x^2+11}\right)$$

$$= \frac{1}{x^2}\left(1+\frac{1}{x^2}\frac{x^4-11x^2}{x^4+3x^2+11}\right) = \frac{1}{x^2}\left(1+\frac{1}{x^2}\frac{(x^4+3x^2+11)-14x^2-11}{x^4+3x^2+11}\right)$$

$$= \frac{1}{x^2}\left(1+\frac{1}{x^2}\left(1-\frac{14x^2+11}{x^4+3x^2+11}\right)\right)$$

Since $\frac{14x^2+11}{x^4+3x^2+11} \geq 0$

$$\frac{x^2+4}{x^4+3x^2+11} = \frac{1}{x^2}\left(1+\frac{1}{x^2}\left(1-\frac{14x^2+11}{x^4+3x^2+11}\right)\right)$$

$$\leq \frac{1}{x^2}\left(1+\frac{1}{x^2}\right) = \frac{1}{x^2}+\frac{1}{x^4} \leq \frac{2}{x^2}$$

for x large