250B
3/27/09

pg. 1

Review from Last Lecture

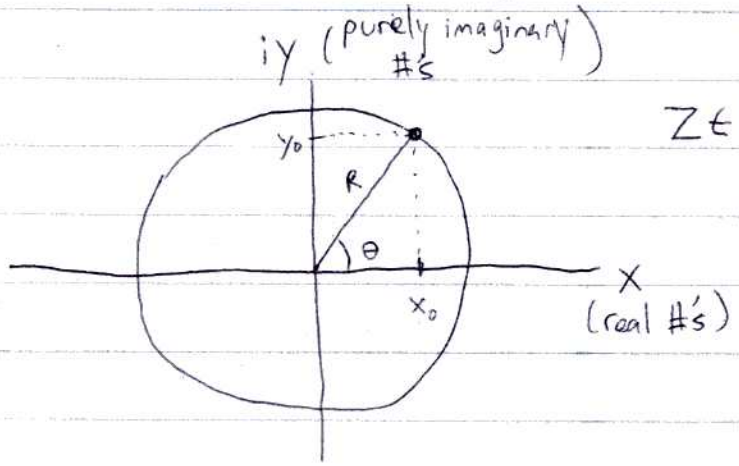
- Solve ODE $y'' + 6y' + 25y = 0$ by assuming solution of basic form $y(x) = e^{\lambda x}$

$\longrightarrow \lambda^2 + 6\lambda + 25 = 0$ (characteristic eqn.)

$\rightarrow$ idea is that you form a linear combination of function based upon solution to characteristic eqn.

- but wait! $\lambda = \dfrac{-6 \pm \sqrt{36-100}}{2} = \dfrac{-6 \pm \sqrt{-64}}{2}$

$= \dfrac{-6 \pm 8i}{2} = -3 \pm 4i$



$z \in \mathbb{C}$

$\boxed{z = x + iy}$ complex #

x, y - real #'s

$R = \sqrt{x_0^2 + y_0^2}$

$\theta = \tan^{-1}\left(\frac{y_0}{x_0}\right)$

$\Longleftrightarrow$

$x_0 = R\cos\theta$

$y_0 = R\sin\theta$

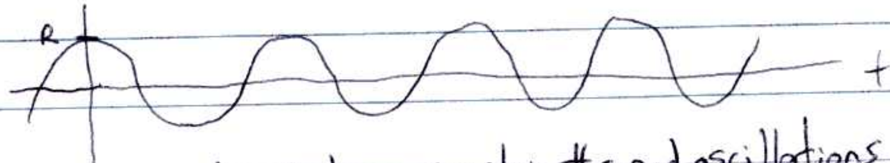
Euler's Formula: $e^{i\theta} = \cos\theta + i\sin\theta$

$$Re^{i\theta} = R\cos\theta + iR\sin\theta$$
$$= x + iy$$

polar form — cartesian form

$\boxed{\text{Oscillations}}$

$f(t)$



→ connection between complex #s and oscillations

sinusoid has three basic properties

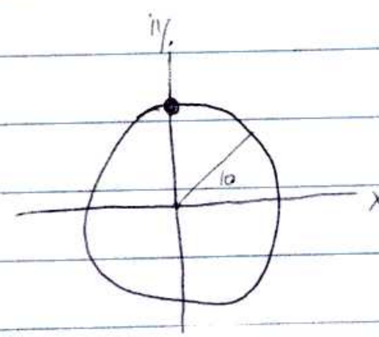
- Ampl. (1)
- period / freq. (2)
- phase (3)

$\boxed{\text{connection back to ODE solutions}}$

$$e^{\lambda t} = e^{(x+iy)t} = \overbrace{e^{xt}}^{\text{amplitude}} \underbrace{e^{iyt}}_{\text{oscillatory term}}$$

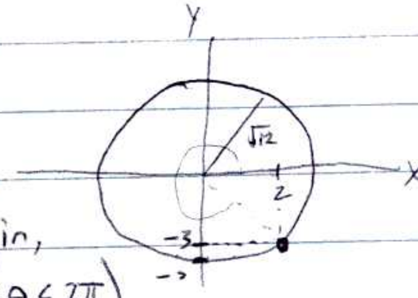
$\boxed{\text{ex}}$ express $10e^{i\pi/2}$ in cartesian coords.

$$10e^{i\pi/2} = 10i$$



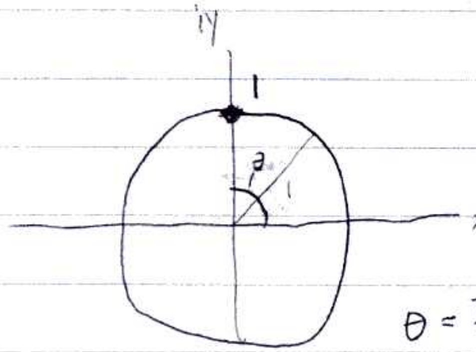
$$x = R\cos\theta = 10\cos\frac{\pi}{2} = 0$$
$$y = R\sin\theta = 10\sin\frac{\pi}{2} = 10$$
$$10e^{i\pi/2} = x + iy = i10$$

$\boxed{\text{ex}}$ $2 - 3i$



→ point will we $\sqrt{12}$ units away from origin, sitting somewhere in fourth quadrant (ie. $\frac{3\pi}{2} \le \theta \le 2\pi$)

ex $i^i = ?$



$\theta = \frac{\pi}{2}$

Consider i in polar form:

$i = 0 + (1)i = (1)e^{i(\frac{\pi}{2})}$

$\longrightarrow i = e^{i\pi/2}$

so $i^i = (e^{i\pi/2})^i = e^{i \cdot i \cdot \pi/2} = e^{-\pi/2}$ (!!)