Other types of linear equations (continued)

Example (continued): $y''' - 5y'' + 12y' - 8y = e^{3t}\cos(2t) + 7te^{t}$

Roots of characteristic equation (from last time):

$\lambda_1 = 1 \qquad \lambda_{2,3} = 2 \pm 2i$

3 linearly independent solutions to the homogeneous equation are:

$y_1(t) = e^{t}\ ;\ y_2(t) = e^{2t}\cos(2t)\ ;\ y_3(t) = e^{2t}\sin(2t)$

General solution to homogeneous equation is

$$y_h(t) = C_1 e^{t} + C_2 e^{2t}\cos(2t) + C_3 e^{2t}\sin(2t)$$

To find a particular solution, use the method of <u>undetermined coefficients.</u>

$$y_p(t) = e^{3t}\left(A\cos(2t) + B\sin(2t)\right) + t(Ct+D)e^{t}$$

Term in $e^{3t}\cos(2t)$: $\alpha = 3$, $\beta = 2$, $3 \pm 2i$ is not a root of the characteristic equation. So try

$$y_{p_1}(t) = e^{3t}\left(A\cos(2t) + B\sin(2t)\right)$$

Term in $7te^{t}$: $\alpha = 1$, $\beta = 0$, 1 is a root of the characteristic equation. So try

$$y_{p_2}(t) = t\,(Ct+D)\,e^{t}$$

Substitute back into ODE & solve for A, B, C, & D:
You will get:

$$A = -\frac{3}{68} \; ; \quad B = \frac{5}{68} \; ; \quad C = \frac{7}{10} \; ; \quad D = \frac{14}{25}$$

So
$$y_p(t) = \frac{-3}{68} e^{3t} \cos(2t) + \frac{5}{68} e^{3t} \sin(2t) + \frac{7}{10} t^2 e^t + \frac{14}{25} t e^t$$

General solution:
$$y(t) = C_1 e^t + C_2 e^{2t} \cos(2t) + C_3 e^{2t} \sin(2t) + y_p(t)$$