

Linear systems

1. Introduction

$$ay'' + by' + cy = 0$$

Let $v = y'$

$$v' = y'' = -\frac{b}{a}y' - \frac{c}{a}y = -\frac{b}{a}v - \frac{c}{a}y$$

Linear system: a system of first-order linear equations

$$\begin{cases} y' = v \\ v' = -\frac{b}{a}v - \frac{c}{a}y \end{cases}$$

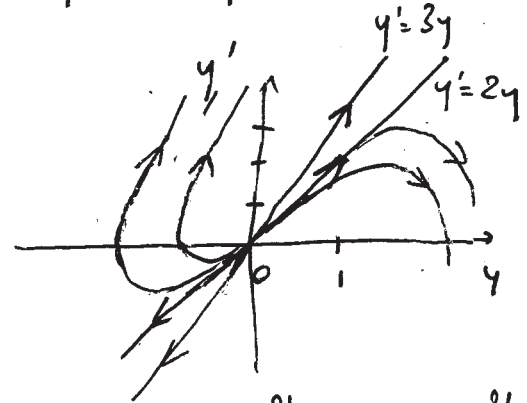
The (y, v) plane is called the phase plane.
We look at trajectories in the phase plane.

Example 1: $y'' - 5y' + 6y = 0$

$$\lambda^2 - 5\lambda + 6 = 0 \Leftrightarrow \lambda = 2, 3$$

$$y = C_1 e^{2t} + C_2 e^{3t}$$

$$y' = 2C_1 e^{2t} + 3C_2 e^{3t}$$



If we choose $C_2 = 0$ we get $y' = 2C_1 e^{2t}$, $y = C_1 e^{2t}$

i.e. $y' = 2y$

If we choose $C_1 = 0$ we get $y' = 3C_2 e^{3t}$, $y = C_2 e^{3t}$

i.e. $y' = 3y$

If C_1 & C_2 are not 0, then

$$\begin{cases} y = C_1 e^{2t} + C_2 e^{3t} \\ y' = 2C_1 e^{2t} + 3C_2 e^{3t} \end{cases}$$