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250B Discussion 3/31

1. • Compute $\frac{2+7i}{4-6i}$

$$\frac{2+7i}{4-6i}\cdot\frac{4+6i}{4+6i}$$

$$\frac{8+12i+28i+42i^2}{16+36}$$

$$\frac{8-42+40i}{52}$$

$$\frac{-34+40i}{52}$$

$$\boxed{\frac{-17+20i}{26}}$$

• Determine i^5

$$i\cdot i\cdot i\cdot i\cdot i$$

$$\sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{-1}$$

$$\sqrt{-1^5}$$

$$\sqrt{-1}$$

$$\boxed{i}$$

2. Use Eulers formula to show:

$$\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$$

$$e^{i(A+B)}=\cos(A+B)+i\sin(A+B)$$

$$e^{iA}e^{iB}=\cos(A+B)+i\sin(A+B)$$

$$(\cos A+i\sin A)(\cos B+i\sin B)=\cos(A+B)+i\sin(A+B)$$

$$\cos A\cos B+i\sin B\cos A+i\sin A\cos B-\sin A\sin B=\cos(A+B)+i\sin(A+B$$

$$[\cos A\cos B-\sin A\sin B]+i[\sin B\cos A+\sin A\cos B]=\cos(A+B)+i\sin(A+B$$

Since the real parts and imaginary parts are equal:

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

-and-

$$\sin(A+B)=\sin B\cos A+\sin A\cos B$$

1. $mx'' + bx' + kx = 0$

$x(t) = Be^{\lambda t}$

$x'(t) = B\lambda e^{\lambda t}$

$x''(t) = B\lambda^2 e^{\lambda t}$

$mB\lambda^2 e^{\lambda t} + bB\lambda e^{\lambda t} + kBe^{\lambda t} = 0$

Simplifying yields

$m\lambda^2 + b\lambda + k = 0$

This is the characteristic equation for the ODE. Solving it for λ yields

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

3. Evaluate: $\int \sin(2x)\cos(4x)\,dx$

First, $\dfrac{e^{i\theta}+e^{-i\theta}}{2} = \dfrac{\cos\theta + i\sin\theta + \cos\theta - i\sin\theta}{2} = \cos\theta$

$\dfrac{e^{i\theta}-e^{-i\theta}}{2i} = \dfrac{\cos\theta + i\sin\theta - \cos\theta + i\sin\theta}{2i} = \sin\theta$

$$\int \frac{e^{i2x}-e^{-i2x}}{2i} \cdot \frac{e^{i4x}+e^{-i4x}}{2}\,dx$$

$$\frac{1}{4i}\int (e^{i2x}-e^{i2x})(e^{i4x}+e^{-i4x})\,dx$$

$$\frac{1}{4i}\int (e^{i6x}-e^{i6x})+(e^{-i2x}-e^{i2x})\,dx$$

8. Show that the magnitude of the product of two complex #'s is the product of the magnitudes. Show that the phase of the product is the sum of the angles.

$r_1 e^{ia}\, r_2 e^{ib} = r_1 r_2 e^{i(a+b)}$

$r_1 r_2 e^{ia+b} = ixe^{ix}$

$xe^{ix} = xe^{iy}$

$x = r_1 r_2$

$y = a+b$

Math 250B Tuesday Notes for March, 31st 2009

1\. Compute $\frac{2+7i}{4-6i} \cdot \frac{(4+6i)}{(4+6i)} \Rightarrow \frac{8+40i+42i^2}{16+36} \Rightarrow$

multiply by congaugate to get real number in denominator

$$\frac{8-42+40i}{52} \Rightarrow \frac{-34+40i}{52} \Rightarrow$$

$$\frac{-17}{26}+\frac{10}{13}i$$

Determine i^5 = (Expand to solve)

$(i \cdot i)\cdot(i \cdot i)\cdot i$

$-1 \cdot -1 \cdot i$

i

4\. Use Euler's Formula to prove De Moivre's Theorem that

$$\boxed{(\cos(\theta)+i\sin(\theta))^n = \cos(n\theta)+i\sin(n\theta)}$$

[Euler's Formula]

$e^{i\theta} = \cos(\theta)+i\sin(\theta)$

$(\cos\theta + i\sin\theta)^n = e^{in\theta}$

Taylor Expansion for $e^{in\theta}$

$$= 1 + in\theta - \frac{(n\theta)^2}{2!} - \frac{i(n\theta)^3}{3!} + \frac{(n\theta)^4}{4!} \dots$$

$$= \left(1 - \frac{(n\theta)^2}{2!} + \frac{(n\theta)^2}{4!} \dots\right) + \left(in\theta - \frac{i(n\theta)^3}{3!} \dots\right)$$

$$= \cos(n\theta) + i\sin(n\theta)$$

5\. Verify that $x(t) = A_1 e^{i\omega t} + A_2 e^{-i\omega t}$ is a solution to the S.H.O $m\ddot{x} + kx = 0$

$mx'' + kx = 0$

$X = A_1 e^{i\omega t} + A_2 e^{-i\omega t}$

$x' = A_1 e^{i\omega t} i\omega A_2 e^{-i\omega t}$

$x'' = \omega^2 A_1 e^{i\omega t} - \omega^2 A_2 e^{-i\omega t}$

$-\omega^2 x$

$m(-w^2x) + Kx = 0$

$K\cancel{x} = mw^2\cancel{x}$

$K = mw^2$

$w = \sqrt{\frac{K}{m}}$

6 Verify Euler's formula: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$e^{ie} = 1 + i\theta + \frac{(i\theta)^2}{2} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \dots$

even Powers: $1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} + \dots = \cos(\theta)$

odd Powers: $i\theta + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^5}{5!}$

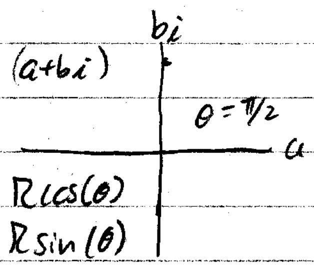
$= i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} + \qquad = \cos\theta$

$= i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}\right) = i\sin(\theta)$

$e^{i\theta} = \cos(\theta) + i\sin\theta$

$\boxed{i = e^{i\pi/2}}$ Polar form

$i = Re^{i\theta}$



$a = R\cos(\theta)$

$b = R\sin(\theta)$