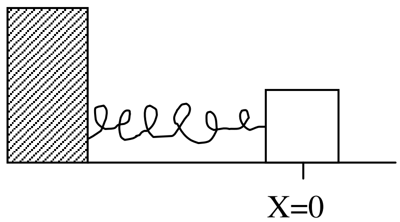
We start with:

$m\ddot{x} + b\dot{x} + kx = 0 \quad m, b, k > 0 \ \dot{x} = \frac{dx}{dt} \ and \ \ddot{x} = \frac{dv}{dt}$

From physics we know F=ma and since a= $\ddot{x}$ this becomes F=m $\ddot{x}$ and in a spring, which is the case we investigated, F=-kx



Looking at the homogeneous case of this would give us:

$m\ddot{x} + kx = 0 \quad and \ x(t) = A\cos(\omega t + \varphi)$

$-\omega^2 Acos(\omega t + \varphi) = -\frac{k}{m} Acos(\omega t + \varphi)$

$\omega = \sqrt{\frac{k}{m}}$ and this is the Resonant frequency

Now we go to the Non-homogeneous case.

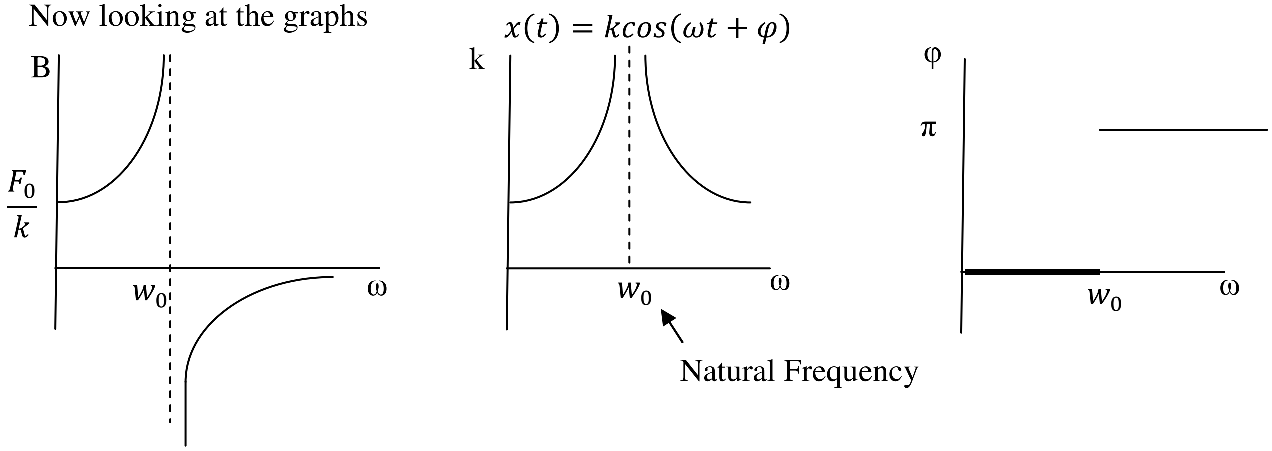
$m\ddot{x} + kx = F(t) = F_0\cos(\omega t) \ but \ this \ time \ x(t) = Bcos(\omega t)$

$-\omega^2 mBcos(\omega t) + kBcos(\omega t) = F_0\cos(\omega t)$

$B(-\omega^2 m + k) = F_0$ the cosines cancel and B factors

$B = \frac{F_0}{-w^2 m + k} = \frac{F_0}{-w^2 m + k} = \frac{F_0}{Bm(\frac{k}{m} - w^2)} = \frac{F_0}{Bm(w_0^2 - w^2)}$

Now looking at the graphs



Now we look again at the equation

$m\ddot{x} + kx = F(t) = F_0\cos(\omega t)\ but\ this\ time\ x(t) = Be^{\lambda t}$

$\lambda = \dfrac{-b \pm \sqrt{b^2 - 4(m)(k)}}{2m}$

Look at the particular solution

$z(t) = Ae^{i(\omega t-\delta)}\ \ \dot{z} = Ai\omega e^{i(\omega t-\delta)}\ \ddot{z}(\mathrm{t}) = -A\omega^2 e^{i(\omega t-\delta)}$

We plug these back into the equation to get

$-\mathrm{m}A\omega^2 e^{i(\omega t-\delta)} + bAi\omega e^{i(\omega t-\delta)} + \mathrm{k}Ae^{i(\omega t-\delta)} = F_0 e^{i\omega t}$

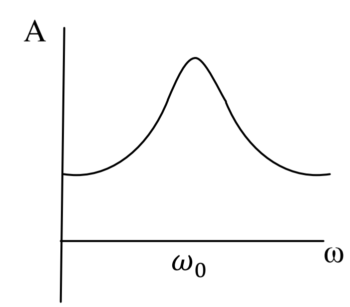
Simplifying gives us the equation

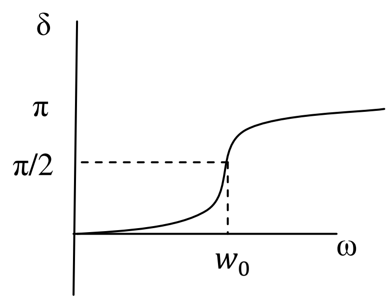
$F_0 = \mathrm{A}e^{-\delta i}[-\mathrm{m}\omega^2 + bi\omega + \mathrm{k}]$

$$A(\omega) = \frac{\frac{F_0}{\mathrm{m}}}{\sqrt{(\omega_0^2 - \omega^2) + (\frac{b\omega}{m})^2}}$$

$$\delta = \tan^{-1}\frac{\frac{b\omega}{m}}{(\omega_0^2-\omega^2)}$$

These are the graphs for the above 2 equations





This all can also be used for understanding of electrical concepts since mechanical and electrical have a very close correlation. Many terms can be related between the two.

Mechanical	**Electrical**
Force	Volts
Velocity	Current
Distance	Charge
μ	Resistance
k	Capacitor
Mass	L (Inductor)