Review Problems for Test 1 - Math 250B

Problem 1. Use slicing to find the volume of a torus (i.e. a tube wrapped around into itself, like a doughnut; see figure below). Use (i) horizontal slices and (ii) vertical shells. From the answer, can you come up with a simple formulation for the volume of a torus?

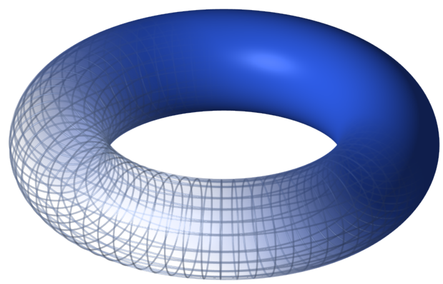

Figure 1: A torus, which is made by revolving a circle of radius r around in a circle of radius R.

Problem 2. Decide whether the following integral converges or diverges. If it converges, give its value.

$$\int_2^\infty \frac{1}{(6-\theta)^2}\, d\theta$$

Problem 3. Calculate the length of the curve described by

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad r = e^{\sin(\theta)} - 2\cos(4\theta) + \sin^5\left(\frac{2\theta-\pi}{24}\right),$$

for $0 \le \theta \le \pi$, correct to 2 decimal places. This curve, called the *butterfly curve*, is shown in the figure below.

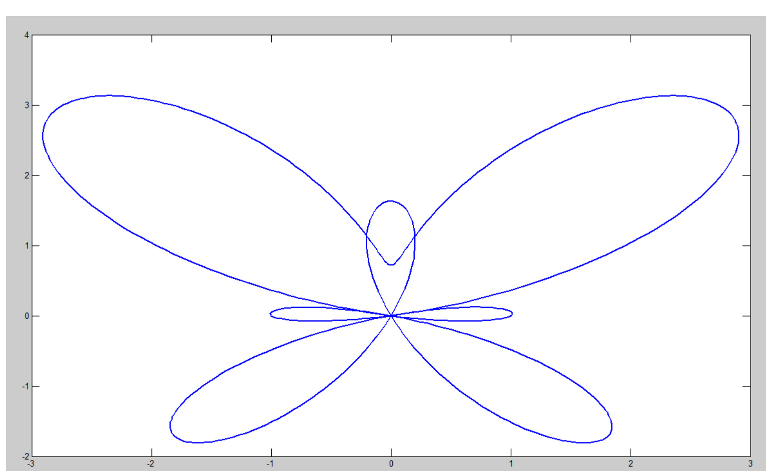

Figure 2: The butterfly curve, for $0 \le \theta \le 2\pi$.