MATH 577 - HOMEWORK PROBLEMS

(1) Consider the system

$$\begin{cases} \dot{A} = B \\ \dot{B} = -\mu A + cB + A^2, \qquad c > 0 \end{cases}$$

(a) For $0 < \mu < c^2/4$, perform a phase plane analysis about the two new fixed points ($A = \pm\sqrt{\mu},\ B = 0$).

(b) Rewrite the nonlinear system in the basis of eigenvectors for the Jacobian of one of the fixed points and choose this fixed point as the new origin in the phase plane.

(c) Perform near-identity changes of variables to eliminate the quadratic terms.

(d) See how the elimination of the quadratic terms affects the cubic terms. Find a simple one-dimensional equation which illustrates this fact.

(2) Check the bifurcation diagrams (solutions & their stability) for the transcritical and for the supercritical pitchfork and Hopf bifurcations.

(3) Consider the two coupled envelope equations describing traveling wave solutions of the complex Swift-Hohenberg equation.

(a) Set $B = 0$ and find a plane wave solutions to the equation for A.

(b) Use the above to find the corresponding expression for the solution $\psi(t)$ of the complex Swift-Hohenberg equation, and compare the result to plane wave solutions of this equation.

(4) Using symmetry arguments, find the amplitude equations describing a perfect hexagonal planform.

(5) Use the above amplitude equations to classify the elementary defects of hexagonal planforms.

(6) Consider the Newell-Whitehead-Segel equation and find the condition for the zig-zag instability of a roll solution.

(7) Consider the complex Ginzburg-Landau equation and find the phase instability condition for plane wave solutions of wavevector k.

(8) Rescale the Kuramoto-Sivashinsky equation and show that the only relevant parameter in this problem is the size of the system.