WRITTEN ASSIGNMENT 2

MATH 223 · SECTION 011 · FALL 2012

1. (a) Two surfaces are called *orthogonal* at a point of intersection if their normal lines are perpendicular at that point. Show that surfaces with equations $F(x,y,z)=0$ and $G(x,y,z)=0$ are orthogonal at an intersection point P where $\nabla F \neq \overrightarrow{0}$ and $\nabla G \neq \overrightarrow{0}$ if and only if $F_xG_x + F_yG_y + F_zG_z = 0$ at P.

(b) Use part (a) to show that the surfaces $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = r^2$ are orthogonal at every point of intersection. Can you see why this is true without using calculus? Try to draw a picture.

2. Suppose that the directional derivatives of $f(x,y)$ are known at a given point in two nonparallel directions given by unit vectors $\overrightarrow{u}$ and $\overrightarrow{v}$. Is it possible to find ∇f at this point? If so, how would you do it?

3. If $z = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

4. problem #28 on page 767.