Section 1.2: Linear Functions and Applications

Preliminary Review

- What is a function $y = f(x)$, if x and y are real numbers?

LINEAR FUNCTION: When two variables are related by a linear equation, with y in terms of x, we say that y is a *linear function* of x, and we write

$$y = f(x) = mx + b.$$

We call x the independent variable and y the dependent variable.

Examples

1. Let $f(x) = 3x - 7$. Find the following:

   (a) $f(5)$

   (b) $f(-4)$

   (c) $f(c + 3)$

   (d) Find x such that $f(x) = 14$.

BREAK-EVEN ANALYSIS: The cost function, $C(q)$, gives the total cost of producing a quantity q of some good. If $C(q)$ is a linear cost function, $C(q) = mq + b$, then

- the fixed costs are represented by the C-intercept (b),
- and the marginal cost is represented by the slope (m).

The revenue function, $R(q)$, gives the total revenue received by a firm from selling a quantity, q, of some good. The profit, $P(q)$, is revenue minus cost. The number of units for which revenue equals cost is called the *break-even quantity.*

Preliminary Exercises

(i) Explain the practical meaning of the *marginal cost*, m.

(ii) If the cost function is $C(x)$ and the revenue function is $R(x)$, where x is the quantity of some good, what is the formula for profit, $P(x)$?

(iii) If $R(x) = 200x$ and $C(x) = 46x + 3500$, determine a simplified formula for $R(x) - C(x)$.

(iv) If a firm sells q units of a good at a price of p, what is the formula for the revenue, $R(q)$?

(v) If a cost function is given by $C(x) = mx + b$, what is the *average cost*? [Note: We will denote average cost by $\overline{C}(x)$.

Examples

2. The manager of a restaurant found that the cost to produce 300 cups of coffee is \$52.05, while the cost to produce 500 cups is \$78.45. Assume the cost $C(x)$ is a linear function of x, the number of cups produced.

   (a) Find a formula for $C(x)$.

   (b) What is the fixed cost?

   (c) Find the total cost of producing 1100 cups.

   (d) Find the marginal cost of a cup of coffee.

   (e) What does the marginal cost of a cup of coffee mean to the manager?

3. A company has a cost function $C(q) = 4000 + 2q$ dollars and revenue function $R(q) = 10q$ dollars.

   (a) What are the fixed costs for the company?

   (b) What is the marginal cost?

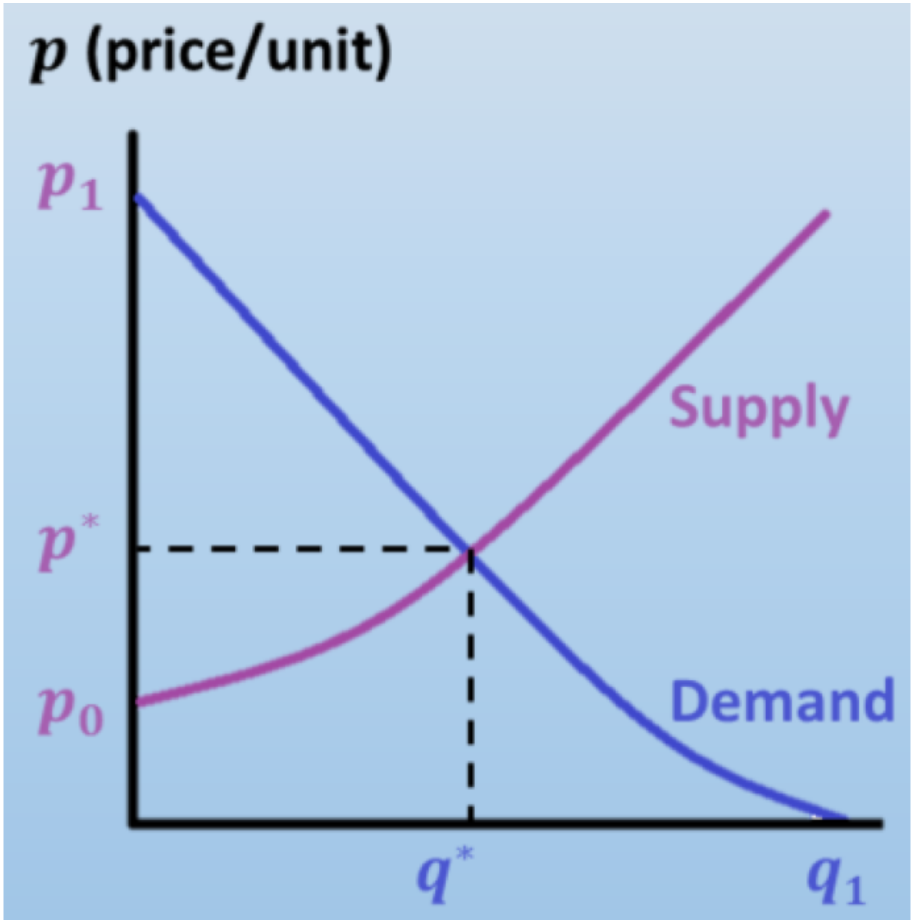
   (c) What price is the company charging for its product?

   (d) Graph $C(q)$ and $R(q)$ on the same axes and label the break-even quantity q_0.

   (e) Find the profit function $P(q)$.

   (f) Find the break-even quantity q_0.

Supply and Demand



- In this course, we will usually write supply and demand curves as price given as a function of quantity: $p = S(q)$, and $p = D(q)$, respectively. However, it usually makes more intuitive sense to think of price as the independent variable and quantity as the dependent variable.

- The equilibrium point (p^*, q^*) (or (p_0, q_0)) is the point where supply and demand are equal, i.e. $S(q^*) = D(q^*)$.

Examples

4. A company has studied the supply and demand for ink pens, and has come up with the following: The quantity demanded (in thousands) is given by the demand function $p = D(q) = 9 - 0.25q$, and the quantity supplied (in thousands) is given by the supply function $p = S(q) = 0.5q$, where the price, p, is measured in dollars.

   (a) Determine the quantity demanded at a price of \$4.

   (b) Determine the quantity supplied at a price of \$4.

   (c) Graph both functions on the same axes. Include all relevant labels.

   (d) Determine the equilibrium quantity and the equilibrium price for the supply and demand system.