

Section 2.1: Properties of Functions

FUNCTION: A function is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the *domain* of the function and the set of resulting output numbers is called the *range* of the function.

VERTICAL LINE TEST: If a vertical line intersects a graph in more than one point, the graph is not the graph of a function.

Examples:

1. Determine whether the following rule defines y as a function of x .

x	7	4	1	0	1	-4	-7
y	49	16	1	0	-1	16	49

Domains of Functions:

We will need to make a distinction in this class between *implied domain* - the set of all possible inputs based on algebraic restrictions - and *contextual domain* - the set of all possible inputs based on the context of the problem.

Examples

2. Find the domain of the function $g(x) = \sqrt{36 - x^2}$.

3. Suppose the revenue function for a given product is $R(q) = -q^2 + 12q$ (where q represents the quantity of the product sold). What is the domain of $R(q)$?

Piecewise Defined Functions:**Examples:**

4. Consider the following piecewise defined function.

$$f(x) = \begin{cases} \frac{2x+4}{x-6} & \text{if } x \neq 6 \\ 4 & \text{if } x = 6 \end{cases}$$

(a) Find $f(6)$.

(b) Find $f(2)$.

(c) Find any value(s) of x such that $f(x) = 1$.

5. Income tax brackets often take the form of piecewise linear functions. Consider the 2010 U.S. Rate Schedule for single persons, showing the income tax owed, T , as a function of adjusted income, i .

$$T = \begin{cases} 0.10i & \text{for } 0 \leq i \leq 8,375 \\ 0.15(i - 8375) + 837.5 & \text{for } 8,375 < i \leq 34,000 \\ 0.25(i - 34000) + 4,681.25 & \text{for } i > 34,000 \end{cases}$$

- (a) If a single person earns an adjusted income of \$30,000, how much income tax would the person owe?

- (b) If a person owes \$3,000 in income tax, what was their adjusted income?