Section 2.2: Quadratic Functions: Translation and Reflection

QUADRATIC FUNCTION: A *quadratic function* is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real numbers with $a \neq 0$.

Review: What kind of shape does the graph of a quadratic function have?

VERTEX OF A QUADRATIC FUNCTION: If a quadratic function is of the form

$$f(x) = ax^2 + bx + c,$$

the *vertex* of the quadratic function is located at the x-value

$$x = -\frac{b}{2a}.$$

The value of the function at the vertex is then given by $f\left(-b/2a\right)$. Writing the vertex as an ordered pair, we then have

$$\text{VERTEX} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Preliminary Exercises:

(i) If $a > 0$, does the vertex of the parabola correspond to a maximum or minimum value of the function?

(ii) If $a < 0$, does the vertex of the parabola correspond to a maximum or minimum value of the function?

Examples:

1. Use translations to graph the function $y = (x + 4)^2 + 5$.

2. Graph the parabola and give its vertex, axis of symmetry, x-intercepts, and y-intercept.
$$y = 2x^2 + 8x - 8.$$

3. Let $C(x) = 2x + 8$ be the cost to produce x batches of widgets, and let $R(x) = -x^2 + 8x$ be the revenue function (each in thousands of dollars).

   (a) Find the minimum break-even quantity.

   (b) Find the maximum revenue.

   (c) Find the maximum profit.

4. A charter flight charges a fare of \$200 per person plus \$4 per person for each unsold seat on the plane (i.e. the fare changes depending on how many unsold seats there are). The plane holds 100 passengers. Let x represent the number of unsold seats.

   (a) Find an expression for the total revenue received for the flight, $R(x)$.

   (b) Graph the expression from above.

   (c) Find the number of unsold seats that will produce the maximum revenue.

   (d) What is the maximum revenue?

   (e) Discuss this pricing model.