Section 2.4: Exponential Functions

THE GENERAL EXPONENTIAL FUNCTION: We say that P is an exponential function of <math display="inline">t with base a if

 $P = P_0 a^t,$

where P_0 is the initial value (when t = 0), and a is the base. We often refer to a as either the growth factor or the decay factor of P. That is, the factor that P changes by when t increases by 1. We have exponential growth when a > 1 and exponential decay when 0 < a < 1. It is worth noting that

we can write

a = 1 + r,

where r is the decimal representation of the percent rate of change (i.e the growth rate or the decay rate). If r > 0, then we have exponential growth and if r < 0, we have exponential decay.

Examples

1. Joe bought a new car for \$30,000 but it depreciates by 15% per year. Write a function for the value of the car over time. Identify the decay factor.

2. The company that produces Cliff Notes (the abridged versions of classic literature) was started in 1958 with \$4000 and sold in 1998 for \$14,000,000. Find the annual percent increase in the value of the company over the 40 years, and write a function for the value of the Cliff Notes company in terms of years since 1958. PROPERTIES OF EXPONENTS: For any integers m and n, and any real numbers a and b, the following properties are satisfied:

1) $a^{m} \cdot a^{n} = a^{m+n}$ 3) $(a^{m})^{n} = a^{mn}$ 5) $\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}$ 7) $a^{-n} = \frac{1}{a^{n}}$ 9) $a^{m/n} = \sqrt[n]{a^{m}} = (\sqrt[n]{a})^{m}$ 2) $\frac{a^{m}}{a^{n}} = a^{m-n}$ 4) $(ab)^{m} = a^{m} \cdot b^{m}$ 6) $a^{0} = 1$ 8) $a^{1/n} = \sqrt[n]{a}$

Examples:

3. Solve for x:

$$3^{2x} = 27$$

4. Solve for x:

$$5^{3-4x} = \frac{1}{625}$$

Interest:

SIMPLE INTEREST: The product of the principal P, the rate r, and time t gives simple interest, I:

I = Prt.

COMPOUND INTEREST: If P dollars is invested at a yearly rate of interest r per year, compounded m times per year for t years, the compound amount (in dollars) is

$$A = P\left(1 + \frac{r}{m}\right)^{mt}.$$

CONTINUOUS COMPOUNDING: If a deposit of P dollars is invested at a rate of interest r compounded continuously for t years, the compound amount (in dollars) is

 $A = Pe^{rt}$.

Examples:

5. Leigh Jacks plans to invest \$500 into a money market account. Find the interest rate that is needed for the money to grow to \$1200 in 14 years if the interest is compounded quarterly.