Section 2.5: Logarithmic Functions

LOGARITHM: For $a > 0$, $a \neq 1$, and $x > 0$,

$$y = \log_a x \text{ means } a^y = x.$$

In other words, $f(x) = \log_a x$ is the inverse to $g(x) = a^x$.

We will see the following types of logarithms the most often:

- **Common Logarithm:** This is the base-10 logarithm, usually denoted simply by log.
$$\log x = \log_{10} x$$

- **Natural Logarithm:** This is the base-e logarithm, usually denoted simply by ln.
$$\ln x = \log_e x.$$

Preliminary Exercises: Evaluate the following without using a calculator.

(i) $\ln 1 =$

(ii) $\log 10 =$

(iii) $\ln\left(\frac{1}{e}\right) =$

(iv) $\log\sqrt{10} =$

(v) $\ln(-10) =$

(vi) $\log 0 =.$

(vii) For $a > 0$, $a \neq 1$, what is the domain of $f(x) = \log_a x$?

(viii) Sketch a graph of $f(x) = \ln x$.

PROPERTIES OF LOGARITHMS: Let x and y be any positive real numbers and r be any real number. If a and b are positive real numbers, $a \neq 1$, $b \neq 1$, then

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a\left(x^r\right) = r\log_a x$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a\left(a^x\right) = x$
7. $a^{\log_a x} = x$
8. $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ *Change of Base Formula*

Examples:

1. Use the properties of logarithms to solve for t:

$$30,000 = 15,000\left(1 + \frac{0.07}{4}\right)^{4t}.$$

4. In the last section, the function that we developed to solve the Cliff Notes problem was $P(t) = 4000(1.226)^t$, where t represented the number of years after 1958. In what year did the value of the Cliff Notes company reach 5 million dollars?