

Section 2.6: Applications: Growth and Decay; Mathematics of Finance

THE CONTINUOUS EXPONENTIAL FUNCTION: If a quantity is said to grow or decay continuously at a rate of k per unit time, then the amount present at time t is given by

$$P = P_0 e^{kt},$$

where P_0 is the initial value (when $t = 0$), and k is the growth constant (if $k > 0$), or decay constant (if $k < 0$).

It will be useful to be able to convert exponential functions of the form $P = P_0 a^t$ into exponential functions of the form $P = P_0 e^{kt}$. In this case

Preliminary Exercises

(i) If $a = e^k$, solve for k .

(ii) If $a > 1$, then what is the sign of k ? If $0 < a < 1$, then what is the sign of k ?

Examples:

1. Suppose a piece of property was worth \$6000 in 2009 and its value is growing by 2.1% per year.

(a) Write a function that models the value of the property over time.

(b) Convert this function to the form $P = P_0 e^{kt}$. Determine the continuous growth rate.

2. Suppose a Treasury Bill (T-Bill) is worth \$500 in 2005 and has a continuous growth rate of 1.7%.

(a) Write a function that models the value of the T-Bill over time.

(b) Convert this function to the form $P = P_0 a^t$. Determine the annual growth rate.

Effective Rate of Compound Interest

Exercise: Suppose that Vanessa Adams invests \$1000 in an account with 6% interest, compounded monthly. After one year, what is the value of the account? By what percentage did the account *actually* grow?

EFFECTIVE RATE OF COMPOUND INTEREST: If r is the annual stated rate, the effective rate of interest is

1. $r_E = \left(1 + \frac{r}{m}\right)^m - 1$ when compounded m times per year.

2. $r_E = e^r - 1$ when compounded continuously.

Examples:

3. Chrintine O'Brien, who is self-employed, wants to invest \$60,000 in a pension plan. One investment offers 8% compounded quarterly. Another offers 7.75% compounded continuously. What is the effective rate in each case?

4. (*Present Value*): Frank Steek must make a balloon payment of \$20,000 in 4 years. Find the present value of the payment if it includes an annual interest of 6.5% compounded continuously.