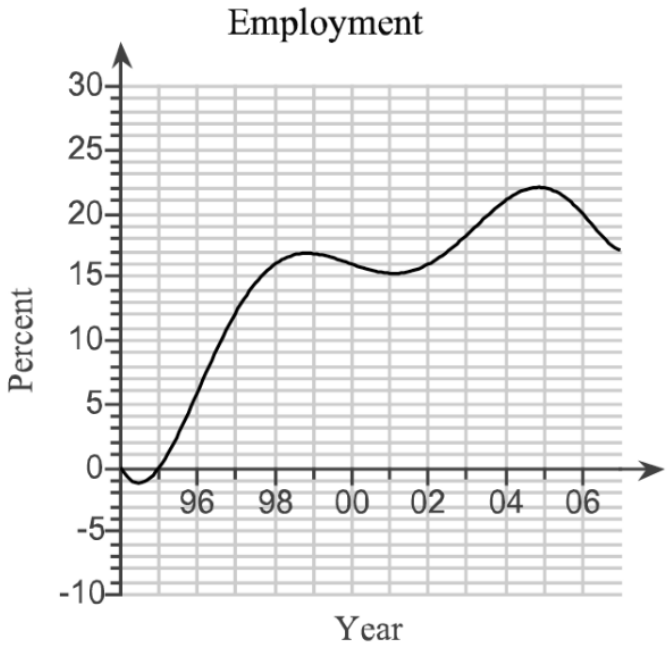
Section 3.3: Rates of Change

AVERAGE RATE OF CHANGE: The *average rate of change* of a function $f(x)$ with respect to x as x changes from $x = a$ to $x = b$ is

$$\text{Average rate of change} = \frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

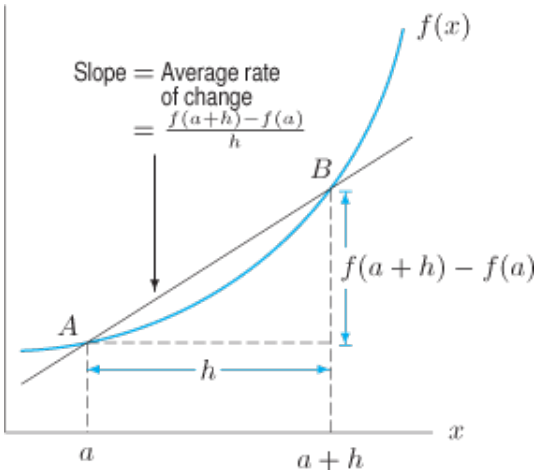
Examples:

1. Use the graph below to estimate the average rate of the percentage of new employees from 1996 to 2005.



2. Find the average rate of change of the function $f(x) = \sqrt{4x + 1}$ between $x = 6$ and $x = 12$.

3. Find a formula for the average rate of change of a function $f(x)$ from $x = a$ to $x = a + h$.



4. Let $f(x) = 3x^2$. Find the average rate of change of $f(x)$ between $x = 1$ and $x = 1 + h$ if

(a) $h = 0.1$

(b) $h = 0.01$

(c) $h = 0.001$

INSTANTANEOUS RATE OF CHANGE: The instantaneous rate of change of for a function f when $x = a$ is

$$\text{Instantaneous rate of change} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

or

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Examples:

5. Find the instantaneous rate of change for the function $f(x) = x^2 + 2x$ when $x = 0$.