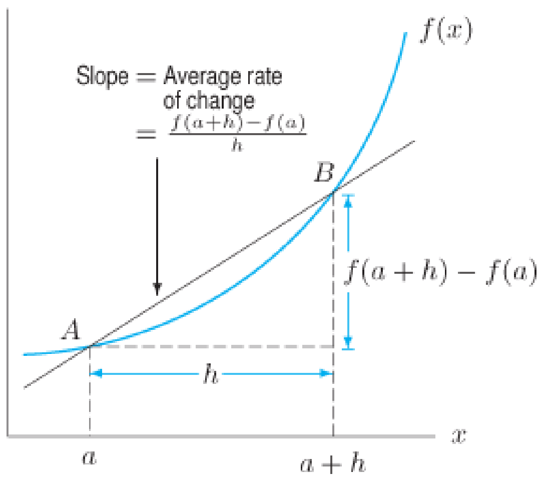
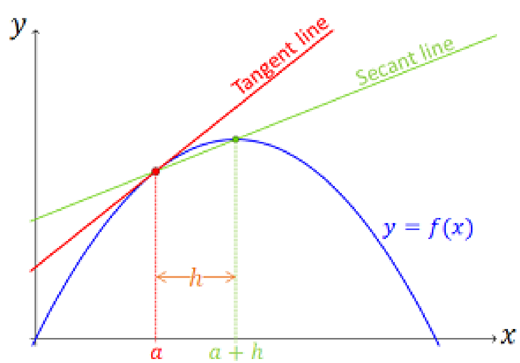
Section 3.4: Definition of the Derivative

THE DERIVATIVE AT A POINT: The *derivative* of a function $f(x)$ at a point $x = a$, written $f'(a)$, is the instantaneous rate of change of $f(x)$ when $x = a$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

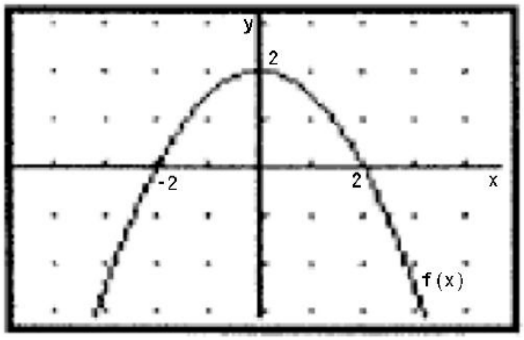
In other words, the derivative is the limit of the average rate of change of $f(x)$ from $x = a$ to $x = a+h$ as h approaches zero. Graphically, this means that $f'(a)$ represents the slope of the *tangent line* to the graph of $f(x)$ at $x = a$.





Examples:

1. The graph of $f(x) = \frac{1}{2}x^2 + 2$ is shown below. Use the graph to determine whether each of the following quantities is positive ("+"), negative ("-"), or zero ("0").



(a) $f(1)$

(b) $f(-3)$

(c) $f(0)$

(d) $f'(1)$

(e) $f'(-3)$

(f) $f'(0)$

EQUATION OF THE TANGENT LINE: The tangent line of the graph of $y = f(x)$ at the point $(a, f(a))$ is given by the equation

$$y = f'(a)(x - a) + f(a),$$

provided $f'(a)$ exists.

3. Find an equation of the tangent line to $f(x) = 6x^2 - 5x - 1$ at the point $(3, 38)$.

4. If $f(x) = x^2 - 3$, find an equation of the tangent line to $f(x)$ at the point $(2, 1)$.

5. Let $f(x) = e^x$. Use a graphing calculator to find $f'(0)$ and $f'(1)$.

THE DERIVATIVE FUNCTION: The *derivative function* of $f(x)$ with respect to x is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

6. Let $f(x) = x^2$. Find $f'(x)$.

7. Let $f(x) = 2x^2 + 3x + 4$. Find $f'(x)$.