Section 4.2: Derivatives of Products and Quotients

The Product Rule:

PRODUCT RULE: If $f(x) = u(x) \cdot v(x)$, and if both $u'(x)$ and $v'(x)$ exist, then

$$f'(x) = u(x) \cdot v'(x) + u'(x) \cdot v(x).$$

In other words,

$$\frac{d}{dx}\left[u(x) \cdot v(x)\right] = u(x) \cdot v'(x) + u'(x) \cdot v(x).$$

Examples:

1. Let $f(x) = x^2(x^3 - 2x)$. Find $f'(x)$ using the product rule.

2. Let $f(x) = x^3(\sqrt{x} + 1)$. Find $f'(x)$ using the product rule.

3. Let $f(x) = (3x^2 + 4)(2x^2 + 3)$. Find $f'(x)$ using the product rule.

4. Let $f(x) = x^2(x + 3)$. Find all values of x where the graph of f has a horizontal tangent line.

The Quotient Rule:

QUOTIENT RULE: If $f(x) = \dfrac{u(x)}{v(x)}$, then

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}.$$

In other words,

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}$$

Examples:

5. If $f(x) = \dfrac{5x^2}{3x+5}$, find $f'(x)$.

6. If $f(x) = \dfrac{x}{x^2+5}$, find $f'(x)$.

7. The total cost (in hundreds of dollars) to produce x units of a product is

$$C(x) = \frac{8x - 4}{5x + 3}.$$

Find $\overline{C}'(x)$, the marginal average cost function.