Section 4.3: The Chain Rule

Review of Composition of Functions:

1. If $f(x) = |x|$, $g(x) = 1 - x^2$, and $h(x) = 2^x$, what is:

   (a) $f(g(5))$

   (b) $g(h(e^3))$

   (c) $f(g(x))$

   (d) $g(h(x))$

   (e) $h(g(x))$

2. Now look at the above problem in reverse. That is, given a function $h(x)$, find functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

   (a) $h(x) = (3x^2 + 1)^3$

   (b) $h(x) = 2e^{5x+1}$

   (c) $h(x) = \sqrt{\ln x}$.

The Chain Rule:

CHAIN RULE: If $h(x) = f(g(x))$, then
$$h'(x) = f'(g(x))g'(x).$$
Using Leibniz notation, if $y = f(g(x))$, let $u = g(x)$. Then $y = f(u)$, and
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Examples:

3. Find the derivatives of the following functions.

   (a) $s = (4t - 1)^5$

   (b) $y = \sqrt{4 - 9x^2}$

(c) $y = \sqrt[3]{(2x^4 + x)^2}$

(d) $g(x) = \left(3x^9 - 17x + \dfrac{1}{x}\right)^2$

(e) $g(x) = \left(3x^4 + 1\right)^4 (x^3 + 4)$

(f) $y = \dfrac{x^2 + 4x}{(3x^3 + 2)^4}$

4. Assume that the total revenue (in dollars) from the sale of x television sets is given by

$$R(x) = 24(x^2 + x)^{2/3}.$$

(a) Find the marginal revenue revenue function, $R'(x)$.

(b) Find and interpret the average revenue from the sale of x sets.

(c) Find and interpret the marginal average revenue.

5. Suppose a demand function is given by

$$q = D(p) = 30\left(5 - \frac{p}{\sqrt{p^2+1}}\right),$$

where q is the demand for a product and p is the price of the item in dollars. Find the rate of change of the demand for the product per unit change in price (i.e. find dq/dp).