## Section 4.4: Derivatives of Exponential Functions

We are interested in knowing how to take the derivative of an exponential function,  $f(x) = a^x$ , where a > 0 and  $a \neq 1$ .

**Question:** What do you expect the derivative of the graph of  $f(x) = a^x$  to look like?

THE EXPONENTIAL RULE: For any positive constant  $a \neq 1$ ,  $\frac{d}{dx}(a^x) = (\ln a) \cdot a^x.$ 

As a special case of the exponential rule, we have a very nice formula for taking the derivative of  $e^x$ :

DERIVATIVE OF  $e^x$ :  $\frac{d}{dx}(e^x) = e^x.$ 

## Examples:

1. Find the derivative of the following functions.

(a) 
$$f(x) = \frac{5^x}{3} - 9e^x$$

(b) 
$$g(t) = 12(0.9)^t$$

Question: What rule do you have to use to take the derivative of a function of the form  $f(x) = a^{f(x)}$ ?

## Examples:

2. Find the derivative of  $f(x) = e^{x^2}$ .

and

$$\frac{d}{dx}(a^{g(x)}) = (\ln a) \cdot a^{g(x)}g'(x)$$
$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x).$$

## Examples:

3. Find the derivative of the following functions:

(a) 
$$g(t) = e^{4t}$$

(b) 
$$g(t) = \frac{1}{e^t}$$

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(c) 
$$h(x) = 10^{5x^2 + 3x + 10}$$

(d)  $h(x) = x4^{-2x}$ 

(e) 
$$h(x) = (3x^2 - 4x)e^{3x}$$

(f) 
$$g(x) = \frac{x^2 e^{2x}}{x + e^{3x}}$$

4. Find an equation for the *tangent line* to  $f(x) = 4\left(\frac{1}{3}\right)^x$  at x = 0.

5. Back in the day, we found that the value of the Cliff Notes company could be modeled by  $P(t) = 4000 (1.226)^t,$ 

where t is the number of years since 1958 and P(t) is the value of the Cliff Notes company in dollars.

(a) Find P'(12) and interpret your answer.

(b) In 1975, how fast is the value of the Cliff Notes company changing?