

Section 4.5: Derivatives of Logarithmic Functions

THE LOGARITHM RULE: For any positive constant $a \neq 1$,

$$\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \frac{1}{x} = \frac{1}{(\ln a)x}.$$

As a special case of the logarithm rule, we obtain a formula for the derivative of $\ln x$:

DERIVATIVE OF $\ln x$:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

Examples:

1. Find the derivative of the following function:

$$g(x) = 3 \ln x + \log x.$$

Question: What rule do you have to use to take the derivative of a function of the form $f(x) = \log_a(g(x))$?

Examples:

2. Find the derivative of $f(x) = e^{x^2}$.

DERIVATIVE OF $a^{g(x)}$ AND $e^{g(x)}$

$$\frac{d}{dx}(a^{g(x)}) = (\ln a) \cdot a^{g(x)} g'(x)$$

and

$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x).$$

Examples:

3. Find the derivative of the following functions:

(a) $g(t) = e^{4t}$

(b) $g(t) = \frac{1}{e^t}$

(c) $h(x) = 10^{5x^2+3x+10}$

(d) $h(x) = x4^{-2x}$

(e) $h(x) = (3x^2 - 4x)e^{3x}$

(f) $g(x) = \frac{x^2 e^{2x}}{x + e^{3x}}$

4. Find an equation for the *tangent line* to $f(x) = 4\left(\frac{1}{3}\right)^x$ at $x = 0$.

5. Back in the day, we found that the value of the Cliff Notes company could be modeled by

$$P(t) = 4000(1.226)^t,$$

where t is the number of years since 1958 and $P(t)$ is the value of the Cliff Notes company in dollars.

- (a) Find $P'(12)$ and interpret your answer.

- (b) In 1975, how fast is the value of the Cliff Notes company changing?