Section 5.1: Increasing and Decreasing Functions

In this section we will begin the process of learning how to apply our knowledge of the derivative in a myriad of ways. To begin, we ask a simple question:

Question: What does the derivative tell us about a function?

THE DERIVATIVE AND DIRECTION: Suppose a function f has a derivative at each point in an open interval. Then

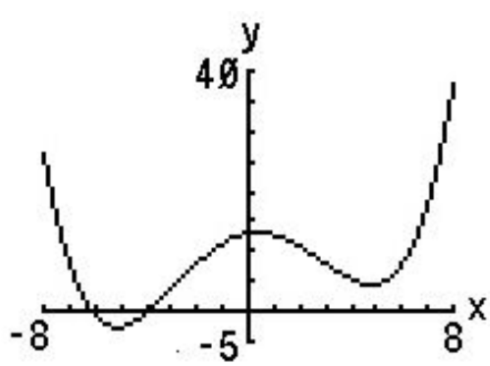
- If $f'(x) > 0$ for each x in the interval, then f is increasing on that interval.
- If $f'(x) < 0$ for each x in the interval, then f is decreasing on that interval.
- If $f'(x) = 0$ for each x in the interval, then f is constant on that interval.

Single x-values where $f'(x) = 0$ are of great interest to us. At these x-values, the tangent line to the graph becomes horizontal, and as a result these could be places where the function attains a maximum or minimum value.

CRITICAL NUMBERS: The *critical numbers* for a function f are those numbers c in the domain of f for which $f'(c) = 0$ or $f'(c)$ is undefined. A *critical point* is an ordered pair whose x-coordinate is a critical number c and whose y-coordinate is $f(c)$.

Examples:

1. Determine the critical number(s) of the function graphed below.



2. Let $f(x) = x^3 - 9x^2 - 48x + 52$.

(a) Find the critical numbers of $f(x)$.

(b) Determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing.

3. For the given functions, find the critical points, and find the intervals where the function increases and the intervals where the function decreases.

   (a) $f(x) = x^4 - 4x^3$.

   (b) $g(x) = \dfrac{x+3}{x-4}$

4. A manufacturer sells video games with the following cost and revenue functions (in dollars), where x is the number of video games sold and $0 \leq x \leq 3300$.

$$C(x) = 0.32x^2 - 0.00004x^3$$
$$R(x) = 0.848x^2 - 0.0002x^3$$

Determine the interval(s) on which the profit function is increasing.

5. The projected year-end assets in the Social Security trust funds, in trillions of dollars, where t represents the number of years since 2000, can be approximated by

$$A(t) = 0.0000329t^3 - 0.00450t^2 + 0.0613t + 2.34,$$

where $0 \leq t \leq 50$. Determine where $A(t)$ is increasing and where $A(t)$ is decreasing.