Section 6.3: Further Business Applications

Elasticity of Demand:

Elasticity of demand, or *demand elasticity*, is designed to give us a quantifiable measure on how sensitive a certain commodity or product is to small fluctuations in price. For example, if the price of a good or service changes by a small fraction, by what fraction does the demand change? How can we quantify this in a nice number that contains all of this information?

Example:

In 2008, the Valve Corporation, a software entertainment company, ran a holiday sale on its popular Steam software program. Using data collected from the sale, it is possible to estimate the demand corresponding to various discounts in the price of the software. Assuming that the original price was \$40, the demand for the software can be estimated by the function

$$q = 3,751,000p^{-2.826},$$

where p is the price and q is the demand. Answer the following questions.

(a) What is the demand when the price is \$40?

(b) Suppose that the price increases by \$2. By what percentage does the price increase by? [Note that the percent increase in price can be thought of as the change in price divided by the original price, $\Delta p/p$.

(c) If the price increases by 5%, by what percent does the demand decrease by?

(d) Compute the ratio

$$\frac{\text{Percent change in demand}}{\text{Percent change in price}}$$

The ratio above is an instance of *demand elasticity*. We can use the tools of calculus to come up with a formula for demand elasticity that corresponds to very tiny fluctuations in price:

Note that the ratio (percentage change in demand)/(percentage change in price) can be written as

$$\begin{aligned}\frac{\text{Percent change in demand}}{\text{Percent change in price}} &= \frac{\Delta q/q}{\Delta p/p} \\ &= \frac{p}{q} \cdot \frac{\Delta q}{\Delta p}\end{aligned}$$

Taking the limit as $\Delta p \to 0$ (tiny fluctuations in price), the ratio $\Delta q/\Delta p$ converges to the derivative dq/dp. Adjusting for the negative sign, we get the following result:

ELASTICITY OF DEMAND: Let $q = D(p)$, where q is the demand at a price of p. The elasticity of demand is

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

- Demand is *inelastic* if $E < 1$
- Demand is *elastic* if $E > 1$
- Demand has *unit elasticity* if $E = 1$.

1. Compute the elasticity of demand from the previous example, where \$40 was the original price and the demand is given by
$$q = 3,751,000p^{-2826}.$$

REVENUE AND ELASTICITY:

1. If the demand is inelastic ($E < 1$), total revenue increases as price increases.
2. If the demand is elastic ($E > 1$), total revenue decreases as price increases.
3. Total revenue is maximized at the price where demand has unit elasticity ($E = 1$).

Examples:

2. Suppose the demand function is given by

$$q = 48,000 - 10p^2.$$

   (a) Find the elasticity, E.

   (b) Find values of p and q (if any) at which total revenue is maximized.

3. Find the elasticity of demand in terms of q if

$$p = 300e^{-0.6q}$$