Section 7.2: Substitution

The goal in this section is to learn how to "undo" the chain rule. Such a process can be very confusing, so let us try to take things one step at a time. First, we will see if we can guess the antiderivative of a function that seems to have obviously come from the chain rule.

Examples:

1. Use the "guess and check" method to find the indefinite integral $\int 2xe^{x^2} dx$.

We will now learn a new way of solving the same problem. This method is more systematic, so that we don't have to be able to "guess" at a solution and get lucky.

2. Find the indefinite integral $\int 2xe^{x^2} dx$ by first performing the substitution $u = x^2$.

What compelled us to make the choice $u = x^2$ in the previous problem? Since we are trying to undo the chain rule, our goal is to find an "inside function", so to speak. There are some patterns that we will learn as we progress in our abilities that will help us make the decision as to what we should set uto be equal to.

Examples:

3. Find the indefinite integral $\int 4x^3 \sqrt{x^4 + 5} \, dx$

4. Discuss the steps of using the substitution method to find an indefinite integral.

5. Find the following indefinite integrals.

(a)
$$\int 10x\sqrt{5x^2+3}\,dx$$

(b)
$$\int \frac{6x^2}{2x^3 - 3} \, dx$$

(c)
$$\int \frac{2x+2}{(x^2+2x+1)^3} dx$$

Let us now switch things up a little bit. In the following examples, it will no longer be the case that the derivative of the inside function perfectly appears next to the outside function. It will still be possible to perform the substitution so long as it is only off by a constant multiple.

6. Find the indefinite integral $\int x^3 \sqrt{x^4 + 5} \, dx$

7. Find the following indefinite integrals.

(a)
$$\int \frac{x}{4x^2 + 1} \, dx$$

(b)
$$\int \frac{1}{3-4x} dx$$

(c)
$$\int x^3 e^{-5x^4} dx$$

8. The rate of growth of profit (in millions of dollars per year) from a new technology is approximated by

$$P'(x) = xe^{-x^2},$$

where x represents the number of years that the new technology has been in operation. Find the total profit function if the total profit in the third year that the new technology has been in operation is \$10,000,000.