Section 8.5: Continuous Money Flow

The tools of calculus provide us a way of analyzing the complexities of money that is flowing into an account on a continuous basis. In all of the following scenarios, we will be assuming that money is flowing at some continuous rate given by a continuous function f(t).

TOTAL MONEY FLOW: If f(t) is the rate of money flow, then the total money flow over the time interval t = 0 to t = T is given by

$$\int_0^T f(t) \, dt$$

It is vital to note the difference between the variable t, and the fixed amount of time, T (a constant).

Examples:

1. The function f(t) = 0.06t + 700 represents the rate of flow of money in dollars per year. Determine the total money flow for the first five years.

2. The rate of continuous money flow starts at \$1000 and increases exponentially at 5% per year for 4 years. Determine the total money flow over that period of time.

We now consider what happens to the formula for present value if we let money flow into an account at a continuous rate, and accumulate compound interest at a continuous rate of r.

PRESENT VALUE OF MONEY FLOW: If f(t) is the rate of continuous money flow at an interest rate r, compounded continuously for T years, then the present value is

$$P = \int_0^T f(t)e^{-rt} dt$$

Examples:

3. The function f(t) = 0.06t + 700 represents the rate of money flow in dollars per year. Determine the present value of the first 5 years if interest earned is 1.2% compounded continuously.

Finally we discuss the accumulated amount of money flow, taking interest into account. We essentially just rewrite the equation $A = Pe^{rT}$ using the formula for P given above.

ACCUMULATED AMOUNT OF MONEY FLOW AT TIME T: If f(t) is the rate of money flow at an interest rate r at time t, the accumulated amount of money flow at time T is

$$A = e^{rT} \int_0^T f(t) e^{-rt} dt$$

Examples:

4. The rate of continuous money flow starts at \$1000 and increases exponentially at 5% per year for 4 years. Find the accumulated amount if interest earned is 3.5% compounded continuously.

5. An investment is expected to yield a uniform continuous rate of money flow of \$20,000 per year for 3 years. Find the total money flow, the present value, and the accumulated amount at an interest rate of 4% compounded continuously.