Math 116 - Spring 2013 Final Exam - Version A

Directions: Please read this cover sheet in its entirety before beginning the exam. Follow the directions carefully. Any violation of the rules stated below will be treated as a potential violation of the Code of Academic Integrity.

- DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
- Write your name, student ID number, and section number at the bottom of this cover sheet.
- On your scantron sheet, blacken in "A" where the sheet says "Test Version" (using a #2 pencil NO PEN). This must be done correctly in order to ensure that your exam is graded properly.
- Your scantron sheet may be pre-printed with the current semester, your name, course, section number, and instructors name. There will be a bar code that represents your student ID. Do not make any marks on the bar code.
- If your scantron sheet does NOT show your name, then, on the scantron sheet (using a #2 pencil NO PEN):
 - Write Spring 2013 beside "Term".
 - Print your first name and last name beside "Name" and include your student ID number.
 - Write Math 116 with the section number beside "Subject, Course, Section".
 - Write your instructor's name beside "Instructor".
- You may use an appropriate calculator on this exam. (TI-83, 84's are fine, TI-89 and other CAS calculators are not). You may not use a cellphone for a calculator. No calculator swapping is permitted do not ask to use another's calculator do not let another use your calculator during this exam. No other electronic devices are permitted during this exam. Please take your calculator out of its case. Your calculator is to be cleared of all memory and programs except possibly the QUADRATIC FORMULA program.
- Please turn off all cell-phones, translators, and other unapproved electronic devices and put them away for the duration of the exam. If you do not know how to turn off your cell-phone, please bring it to the front desk.
- Please do all of your work in this test booklet.
- You should only have on your desk: your CatCard, writing utensils/erasers, and an appropriate calculator (out of its case). All other belongings should be packed neatly in your backpack/bag.
- In the multiple choice section, blacken the corresponding circles to your answers on the scantron sheet provided. If you want to change your answers on the scantron sheet, erase completely do not X-out the wrong answer. As you put your answers on the scantron sheet, also circle your answers in this test booklet. No partial credit will be given in this section. Check to be sure your test contains 24 multiple choice questions.
- In the short answer section, be sure to show all work, define all variables, label all units, and label all graphs. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit. Check to be sure your test contains 3 short answer questions.
- When you are done, turn in both your answer sheet and test booklet.
- You may NOT leave the examination room before 9:30 p.m.

I have read, and I understand, these instructions.

(Printed Name)

(Signature)

(Student ID Number)

(Section Number)

MULTIPLE CHOICE: Indicate your answer on the Scantron sheet provided and circle your answer in this test booklet. No partial credit will be given in this section.

1. The Monkey Wrench Company spends \$7000 to produce 110 parts, achieving a marginal cost of \$40. Find the linear cost function.

(A) C(x) = 40x + 2600 (B) C(x) = 110x + 7000 (C) C(x) = 110x + 40

(D) C(x) = 40x + 7000 **(E)** none of these

2. Find the slope of a line that goes through the points (-5, 8) and (7, 2). The slope is

- (A) less than -1 (B) between -1 and 1 (C) between 1 and 3
- (D) more than 3 (E) undefined

3. Suppose that the total profit in hundreds of dollars from selling x items is given by $P(x) = 3x^2 - 16x + 7$. Find the marginal profit at x = 4.

The marginal profit is

- (A) less than 650 per item
- (B) between 650 and 750 per item

 (\mathbf{D}) between \$850 and \$950 per item

- (C) between \$750 and \$850 per item
- (E) more than \$950 per item

4. Jeremy Clark must make a balloon payment of \$25,000 in 5 years. Find the present value of the payment if it includes annual interest of 3.5% compounded daily.

The present value is

(A) less than \$14,000

(B) between \$14,000 and \$17,000

(C) between \$17,000 and \$20,000

(D) between \$20,000 and \$23,000

(E) more than \$23,000

5. Find the effective rate corresponding to a nominal rate of 7.9% compounded monthly. The effective rate is

(A) less than 7.9%	(B) between 7.9% and 8.1%	(C) between 8.1% and 8.3%

(D) between 8.3% and 8.5% (E) more than 8.5%

6. For $f(x) = x^2 + 4x + 2$, find the equation of the tangent line when x = -3. The tangent line is

(A) $y = -2(x+3) - 1$	(B) $y = -2(x-3) - 19$	(C) $y = -3(x+2) - 19$
(D) $y = -3(x-2) - 1$	(E) none of these	

7. Use the graph of $f(x) = 5e^{-x^2}$ below to find $\lim_{x \to \infty} 5e^{-x^2}$.



8. Let C(x) = 5x + 21 be the cost to produce x widgets, and let $R(x) = -x^2 + 15x$ be the revenue. Find the minimum break-even quantity.

The minimum break-even quantity is

(A) less than 0.5	(B) between 0.5 and 1.5	(C) between 1.5 and 2.5
(D) between 2.5 and 3.5	(E) more than 3.5	

9. Assume that a demand equation is given by q = 24,000 - 200p. Find the marginal revenue for the production level q = 1000 units.

The marginal revenue at 1000 units is

(\mathbf{A}) less than 25	(B) between 25 and 50	(C) between 50 and 75 $($

(D) between 75 and 100 (E) more than 100

10. Determine the critical number(s) of the function graphed below.



(C) x = 0 only

(D) x = 4 only

 $({\bf E})$ none of these

- 11. Suppose that f(x) and g(x) are differentiable functions such that f(5) = -3, f'(5) = 5, g(5) = 6, and g'(5) = 2. Find h'(5) when $h(x) = f(x) \cdot g(x)$. h'(5) is
 - (A) less than 0 (B) between 0 and 7 (C) between 7 and 14
 - (**D**) between 14 and 21 (**E**) more than 21

12. Find the elasticity of demand for the demand function

 $q = 1130p^{-1.05}$

for p = \$30.

The elasticity of demand for p = \$30 is

(A) less than 0.5 (B) between 0.5 and 1.0 (C) between 1.0 and 2.0

(D) between 2.0 and 4.0 (E) more than 4.0

13. A restaurant has an annual demand for 2554 bottles of California wine. It costs \$3 to store 1 bottle for 1 year, and it costs \$7 to place a reorder. Find the optimum number of bottles per order.

The optimum number of bottles per order is

(\mathbf{A}) less than 105	(B) between 105 and 115	(C) between 115 and 125
(\mathbf{D}) between 125 and 135	(E) more than 135	

14. Consider the following table of values of the functions f and g and their derivatives at various points.

x	1	2	3	4
f(x)	3	4	1	2
f'(x)	-4	-6	8	9
g(x)	3	4	2	1
g'(x)	5/9	4/9	8/9	1

Find
$$\frac{d}{dx}[f(g(x))]$$
 at $x = 2$.

 (\mathbf{A}) less than -6

(B) between -6 and -2

 (\mathbf{C}) between -2 and 2

(D) between 2 and 6 (E) more than 6

15. Suppose that the graph below is the graph of f'(x), the derivative of a function f(x). Find the locations of all relative extrema of f(x), and tell whether each is a relative maximum or minimum.



- (A) x = -3 gives a relative maximum & x = 1 gives a relative minimum



- (C) x = -1 gives a relative maximum
- (E) none of these

(D) x = -1 gives a relative minimum

16. A manufacturer sells video games with the following cost and revenue functions (in dollars), where x is the number of games sold.

$$C(x) = 1.42x^{2} - 0.0038x^{3}$$
$$R(x) = 4.39x^{2} - 0.0049x^{3}$$

Determine the interval on which the profit function is increasing.

(A) (0,2700) (B) $(0,\infty)$ (C) (0,3600) (D) $(900,\infty)$ (E) none of these

17. If we use the substitution $u = 2 + e^{4x}$, then which of the following is equivalent to

$$\int \frac{4e^{4x}}{2+e^{4x}} dx.$$
(A) $\int \frac{1}{u} du$ (B) $\int \frac{u-2}{u} du$ (C) $\int \frac{4}{u} du$ (D) $\int \frac{4(u-2)}{u} du$ (E) none of these

c

18. The graph below shows a city's daily rate of use of electricity (in millions of kilowatts). Estimate the total daily usage of electricity by summing the area of rectangles. Use the left endpoints, then the right endpoints, then give the average of those results as the answer. Let the width of each rectangle be 6 hours.



The estimate of the city's total daily usage of electricity for the given day, found by averaging the estimates using left and right endpoints, is:

- (A) less than 100 million kw-hrs
- (B) between 100 million kw-hrs and 140 million kw-hrs
- (C) between 140 million kw-hrs and 180 million kw-hrs
- (D) between 180 million kw-hrs and 220 million kw-hrs
- (E) more than 220 million kw-hrs

19. Find the exact value of the integral using formulas from geometry.

(A)
$$64\pi$$
 (B) 32π (C) 16π (D) 8π (E) none of these

20. Find the cost function if the marginal cost function is

$$C'(x) = 8x - 7$$

and the fixed cost is 18.

(A) $C(x) = 4x^2 - 7x + 18$ (B) C(x) = 8x + 11 (C) C(x) = 18x + 18(D) $C(x) = 4x^2 + 11x$ (E) none of these 21. Find the total area of the shaded regions.



The total area of the shaded regions is

(A) less than 45
(B) between 45 and 50
(C) between 50 and 55
(D) between 55 and 60
(E) more than 60

22. A small company of science writers found that its rate of profit (in dollars) after t years of operation is given by the function below.

$$P'(t) = 200(t+2)\left(t^2 + 4t + 6\right)^{\frac{1}{3}}.$$

Find the total profit in the first four years.

The total profit is

- (A) less than \$7000 (B) between \$7000 and \$7500 (C) between \$7500 and \$8000
- (D) between \$8000 and \$8500 (E) more than \$8500

23. Find the producers' surplus if the supply function for pork bellies is given by

$$S(q) = q^{3/2} + 3q^{1/2} + 63.$$

Assume the supply and demand are in equilibrium at q = 25. The producers' surplus is

(A) less than \$1800 (B) between \$1800 and \$2200 (C) between \$2200 and \$2400

(D) between \$2400 and \$2800 (E) more than \$2800

24. The rate of a continuous money flow starts at \$1400 and increases exponentially at 2% per year for 5 years. Find the present value if interest earned is 6% compounded continuously.

The present value is

(A) less than \$5200 (B) between \$5200 and \$5600 (C) between \$5600 and \$6000

(D) between \$6000 and \$6400 (E) more than \$6400

SHORT ANSWER: Be sure to show all work, define all variables, label all units. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit.

- 1. The manager of an 80-unit apartment complex is trying to decide what rent to charge. Experience has shown that at a rent of \$700, all the units will be full. On the average, one additional unit will remain vacant for each \$25 increase in rent.
 - (a) Let x represent the number of \$25 increases. Find an expression for the total revenue from all rented apartments.

(b) What value of x leads to maximum revenue?

(c) What is the maximum revenue?

2. For the cost and price functions

$$C(q) = 212 + 22q; \quad p = 112 - 3q,$$

find:

(a) the number, q, of units that produces maximum profit;

(b) the maximum profit.

3. The supply function for oil is given (in dollars) by S(q), and the demand function is given (in dollars) by D(q):

$$S(q) = q^2 + 7q;$$
 $D(q) = 1102 - 13q - q^2.$

(a) Graph the supply and demand curves on the same axes.

(b) Find the point at which supply and demand are in equilibrium.

(c) Find the consumers' surplus.

(d) Find the producers' surplus.

Formulas You Might Find Useful

Ι	=	Prt	Α	=	$P\left(1+\frac{r}{m}\right)^{mt}$
A	=	Pe^{rt}	r_E	=	$\left(1+\frac{r}{m}\right)^m - 1$
r_E	=	$e^r - 1$	f'(x)	=	$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx}\left[f(g(x))\right]$	=	$f'(g(x))\cdot g'(x)$	$\frac{d}{dx}\left[u(x)\cdot v(x)\right]$	=	$u(x) \cdot v'(x) + v(x) \cdot u'(x)$
$\frac{d}{dx} \left[\frac{u(x)}{v(x)} \right]$	=	$\frac{v(x)\cdot u'(x)-u(x)\cdot v'(x)}{[v(x)]^2}$	$\frac{d}{dx}\left[a^{x}\right]$	=	$(\ln a)a^x$
$\frac{d}{dx}\left[\log_a(x)\right]$	=	$\frac{1}{(\ln a)x}$	E	=	$-rac{p}{q}\cdot rac{dq}{dp}$
q	=	$\sqrt{\frac{2fM}{k}}$	$\int x^n dx$	=	$\frac{x^{n+1}}{n+1} + C$
$\int a^{kx} dx$	=	$\frac{a^{kx}}{k(\ln(a))} + C$	$\int x^{-1} dx$	=	$\ln x + C$
C.S.	=	$\int_0^{q_0} \left(D(q) - p_0 \right) dq$	<i>P.S.</i>	=	$\int_0^{q_0} \left(p_0 - S(q) \right) dq$
Р	=	$\int_0^T f(t)e^{-rt}dt$	A	=	$e^{rT} \int_0^T f(t) e^{-rt} dt$