## Math 116 - Spring 2013 Final Exam Solutions - Version A

MULTIPLE CHOICE: Indicate your answer on the Scantron sheet provided and circle your answer in this test booklet. No partial credit will be given in this section.

#1 A	$\#7 \mathrm{B}$	#13  B	#19 C
#2 B	#8 D	#14 D	#20 A
$\#3~\mathrm{C}$	#9 E	$\#15 \mathrm{~B}$	#21 A
#4 D	#10 D	#16 E	$\#22 \ \mathrm{E}$
$\#5~\mathrm{C}$	#11 E	#17 A	$\#23 \mathrm{~B}$
#6 A	$\#12 \mathrm{~C}$	#18 C	#24 D

SHORT ANSWER: Be sure to show all work, define all variables, label all units. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit.

- 1. The manager of an 80-unit apartment complex is trying to decide what rent to charge. Experience has shown that at a rent of \$700, all the units will be full. On the average, one additional unit will remain vacant for each \$25 increase in rent.
  - (a) (4 points) Let x represent the number of \$25 increases. Find an expression for the total revenue from all rented apartments.

Rent: (700 + 25x)Apartments Occupied: (80 - x)Total Revenue: R(x) = (700 + 25x)(80 - x)or  $R(x) = -25x^2 + 1300x + 56000$ .

(b) (4 points) What value of x leads to maximum revenue?

We will find the maximum by 1st: finding all critical points 2nd: testing the critical points and endpoints.

We find critical points by finding R'(x) and determining where it is zero or undefined. R'(x) = -50x + 1300R'(x) is never undefined, and is zero when x = 26.

From the context, we see that a maximum cannot occur if x is less than zero, nor if x is more than 80.

Now we test the critical point and endpoints: R(0) = 56,000, R(26) = 72,900, R(80) = 0 so the absolute maximum occurs at x = 26.

(c) (2 points) What is the maximum revenue?

The maximum revenue is \$72,900.

2. For the cost and price functions

$$C(q) = 212 + 22q; \quad p = 112 - 3q,$$

find:

(a) (8 points) the number, q, of units that produces maximum profit;

Revenue:  $R = p \cdot q = (112 - 3q)q = 112q - 3q^2$ 

Profit: 
$$P(q) = R(q) - C(q)$$
  
 $P(q) = 112q - 3q^2 - (212 + 22q).$   
 $P(q) = -3q^2 + 90q - 212.$ 

We will find the maximum by 1st: finding all critical points 2nd: testing the critical points and endpoints.

We find critical points by finding P'(q) and determining where it is zero or undefined. P'(q) = -6q + 90P'(q) is never undefined, and is zero when q = 15.

From the context, we see that a maximum cannot occur if q is less than zero, nor if q is more than  $37\frac{1}{3}$ .

Now we test the critical point and endpoints: P(0) = -212, P(15) = 463,  $P(37\frac{1}{3}) = -1033\frac{1}{3}$ so the absolute maximum occurs at q = 15.

(b) (2 points) the maximum profit.

The maximum profit is 463.

3. The supply function for oil is given (in dollars) by S(q), and the demand function is given (in dollars) by D(q):

$$S(q) = q^2 + 7q;$$
  $D(q) = 1102 - 13q - q^2.$ 

(a) (2 points) Graph the supply and demand curves on the same axes.



(b) (2 points) Find the point at which supply and demand are in equilibrium.

Supply and demand are in equilibrium when S(q) = D(q)  $q^2 + 7q = 1102 - 13q - q^2$   $2q^2 + 20q - 1102 = 0$  2(q - 19)(q + 29) = 0 q = 19 or q = -29Since q = -29 makes no sense in context, and D(19) = S(19) = 494, our equilibrium solution is q = 19, p = 494.

(c) (3 points) Find the consumers' surplus.

$$C.S. = \int_{0}^{q_0} (D(q) - p_0) dq$$
  
=  $\int_{0}^{19} ((1102 - 13q - q^2) - 494) dq = $6919.17$ 

(d) (3 points) Find the producers' surplus.

$$P.S. = \int_{0}^{q_0} (p_0 - S(q)) dq$$
  
=  $\int_{0}^{19} (494 - (q^2 + 7q)) dq = $5836.17$