## Math 116 - Spring 2014 Final Exam - Version A

**Directions**: Please read this cover sheet in its entirety before beginning the exam. Follow the directions carefully. Any violation of the rules stated below will be treated as a potential violation of the Code of Academic Integrity.

- DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
- Write your name, student ID number, and section number at the bottom of this cover sheet.
- On your scantron sheet, blacken in "A" where the sheet says "Test Version" (using a #2 pencil NO PEN). This must be done correctly in order to ensure that your exam is graded properly.
- Your scantron sheet may be pre-printed with the current semester, your name, course, section number, and instructors name. There will be a bar code that represents your student ID. Do not make any marks on the bar code.
- If your scantron sheet does NOT show your name, then, on the scantron sheet (using a #2 pencil NO PEN):
  - Write Spring 2014 beside "Term".
  - Print your first name and last name beside "Name" and include your student ID number.
  - Write Math 116 with the section number beside "Subject, Course, Section".
  - Write your instructor's name beside "Instructor".
- You may use an appropriate calculator on this exam. (TI-83, 84's are fine, TI-89 and other CAS calculators are not). You may not use a cellphone for a calculator. No calculator swapping is permitted do not ask to use another's calculator do not let another use your calculator during this exam. No other electronic devices are permitted during this exam. Please take your calculator out of its case. Your calculator is to be cleared of all memory and programs except possibly the QUADRATIC FORMULA program.
- Please turn off all cell-phones, translators, and other unapproved electronic devices and put them away for the duration of the exam. If you do not know how to turn off your cell-phone, please bring it to the front desk.
- Please do all of your work in this test booklet.
- You should only have on your desk: your CatCard, writing utensils/erasers, and an appropriate calculator (out of its case). All other belongings should be packed neatly in your backpack/bag.
- In the multiple choice section, blacken the corresponding circles to your answers on the scantron sheet provided. If you want to change your answers on the scantron sheet, erase completely do not X-out the wrong answer. As you put your answers on the scantron sheet, also circle your answers in this test booklet. No partial credit will be given in this section. After you are allowed to open your exam, check to be sure your test contains 24 multiple choice questions.
- In the short answer section, be sure to show all work, define all variables, label all units, and label all graphs. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit. After you are allowed to open your exam, check to be sure your test contains 3 short answer questions.
- When you are done, turn in both your answer sheet and test booklet.
- You may NOT leave the examination room before 9:30 p.m.

I have read, and I understand, these instructions.

(Printed Name)

(Signature)

(Student ID Number)

(Section Number)

MULTIPLE CHOICE: Indicate your answer on the Scantron sheet provided and circle your answer in this test booklet. No partial credit will be given in this section.

1. Determine the equation of the line given below.



2. The manager of a restaurant found that the cost to produce 200 cups of coffee is \$71.25, while the cost to produce 500 cups is \$128.25. Assume the cost C(x) is a linear function of x, the number of cups produced. Find the total cost of producing 900 cups. The total cost is

(A) less than \$200	<b>(B)</b> between \$200 and \$210	(C) between \$210 and \$220
( <b>D</b> ) between \$220 and \$230	(E) more than \$230	

- 3. A product has a production cost function C(x) = 385x + 7290 and a revenue function R(x) = 520x. Find the break-even quantity. The break-even quantity is
  - (A) between 40 units and 45 units (B) between 45 units and 50 units
  - (C) between 50 units and 55 units (D) between 55 units and 60 units
  - (E) between 60 units and 65 units

4. Let

$$f(x) = \begin{cases} \frac{3x-5}{x+1} & \text{if } x \neq -1 \\ -2 & \text{if } x = -1 \end{cases}$$

Find the value of x such that f(x) = 2.

(A) less than -5 (B) between -5 and 0 (C) between 0 and 5

(D) between 5 and 10 (E) more than 10

- 5. Find the effective rate corresponding to a nominal rate of 7.9% compounded continuously. The effective rate is
  - (A) less than 8.2% (B) between 8.2% and 8.3% (C) between 8.3% and 8.4%
  - (D) between 8.4% and 8.5% (E) more than 8.5%

6. The graph of  $f(x) = e^{-x} + 2$  is given below.



Use the graph to find  $\lim_{x \to \infty} f(x)$ , if it exists.

(A) the limit does not exist (B) c	$\infty$ (C) 5 (D)	3 (E) 2
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7. Consider the function shown in the graph below.



Choose the interval(s) on which the *rate of change* is negative.

(A) $(-\infty, 2)$ and $(2, 5)$	<b>(B)</b> (2,5) only	(C) $(5,\infty)$ only
(D) $(4,\infty)$ only	<b>(E)</b> (2,4) only	

8. Suppose the demand for a certain item is given by  $D(p) = -5p^2 - 7p + 400$ , where p represents the price of the item in dollars. Find the rate of change of demand with respect to price. The rate of change with respect to price is

(A) 
$$-10p - 7$$
 (B)  $-5p^2 - 7p + 400$  (C)  $-5p$   
(D)  $-5p + 393$  (E)  $-\frac{5p^3}{3} - \frac{7p^2}{2} + 400p$ 

9. Find the derivative of the function  $g(x) = -8x^3 + \frac{7}{x^5}$ .

(A) 
$$g'(x) = -24x^2 - \frac{35}{x^6}$$
 (B)  $g'(x) = -24x^2 + \frac{35}{x^4}$  (C)  $g'(x) = -2x^4 - \frac{7}{6x^6}$   
(D)  $g'(x) = -24x^4 - \frac{35}{x^4}$  (E)  $g'(x) = -2x^4 + \frac{7}{6x^6}$ 

10. Consider the following table of values of the function g and its derivatives at various points.

x	1	2	3	4
g(x)	3	4	4	1
g'(x)	5/3	-3/2	-5/3	-5/2

Find 
$$\frac{d}{dx} [x \cdot g(x)]$$
 at  $x = 2$ . The value of  $\frac{d}{dx} [x \cdot g(x)]$  at  $x = 2$  is  
(A) between -4 and -2
(B) between -2 and 0
(C) between 0 and 2
(D) between 0 and 2

(D) between 2 and 4 (E) between 4 and 6

11. Using data in a car magazine, we constructed the mathematical model

$$y = 100e^{-0.0901t}$$

for the percent of cars of a certain type still on the road after t years. Find the rate of change of the percent of cars still on the road after 5 years.

The rate of change is

(A) less than $-7\%$ per year	(B) between $-7\%$ and $-5\%$ per year
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- (C) between -5% and -3% per year
- (D) between -3% and -1% per year

(E) more than -1% per year

12. The cost (in dollars) of producing q headphones is given by

$$C(q) = 4q^2 - 6q + 100$$

Identify the open interval where the average cost  $\overline{C}(q)$  is increasing.

(A) 
$$(5,\infty)$$
 (B)  $(0,5)$  (C)  $(0,\infty)$  (D)  $\left(\frac{3}{4},\infty\right)$  (E) None of these

13. Let f(x) be a continuous function with two critical points whose derivative has the following values:

x	3	4	5	6	7
f'(x)	3	0	-2	0	1

f(x) has a relative minimum at

(A) x = 3 (B) x = 4 (C) x = 5 (D) x = 6 (E) None of these

14. Find the absolute minimum value of the function

$$f(x) = x^3 + 7x^2 - 5x - 20$$

over the interval [-1, 4].

The absolute minimum occurs at

(A) 
$$x = -1$$
 (B)  $x = \frac{1}{3}$  (C)  $x = -5$  (D)  $x = 4$  (E) None of these

15. Comfy Chair Company has an annual demand for 26,000 reclining chairs. It costs \$17 to store 1 reclining chair for 1 year, and \$2200 in set up cost to produce each batch. Find the number of reclining chairs per batch that should be produced to minimize cost.

The optimum number of reclining chairs per batch is

(A) between 2100 and 2300	<b>(B)</b> between 2300 and 2500	(C) between 2500 and 2700
<b>(D)</b> between 2700 and 2900	<b>(E)</b> between 2900 and 3100	

16. The short-term demand for mozzarella cheese in the U.S. in 2011 can be approximated by

 $q = f(p) = 33e^{-0.11p},$ 

where p represents the price of mozzarella cheese in dollars per pound and q represents the per capita consumption of mozzarella cheese. What is the elasticity of demand for mozzarella cheese when the price is \$10 per pound?

The elasticity of demand is

- (A) between 0.4 and 0.6 (B) between 0.6 and 0.8 (C) between 0.8 and 1.0
- (D) between 1.0 and 1.2 (E) between 1.2 and 1.4

17. Evaluate the following indefinite integral.

$$\int (e^{5x} + 4x^3) dx$$
(A)  $5e^{5x} + 12x^2 + C$ 
(B)  $5xe^{5x-1} + 12x^2 + C$ 
(C)  $\frac{e^{5x}}{5} + x^4 + C$ 
(D)  $\frac{e^{5x+1}}{5x+1} + x^4 + C$ 
(E) None of these

18. Find the revenue function if the marginal revenue function is

$$R'(x) = -0.4x^3 + 36x^2 + 25.$$

Recall that if no items are sold, the revenue is 0.

(A)  $R(x) = -1.2x^3 + 72x^2 + 25$ (B)  $R(x) = -0.1x^3 + 12x^2 + 25$ (C)  $R(x) = -1.2x^2 + 72x$ (D)  $R(x) = -0.4x^4 + 36x^3 + 25x$ 

(E) 
$$R(x) = -0.1x^4 + 12x^3 + 25x$$

19. If we use the substitution  $u = x^3 - 9$ , then which of the following is equivalent to

$$\int x^{2}(x^{3}-9)^{1/5}dx?$$
(A)  $\int \frac{u^{1/5}}{3}du$ 
(B)  $\int u^{1/5}du$ 
(C)  $\int (u+9)^{2/3}u^{1/5}du$ 
(D)  $\int \frac{(u+9)^{2/3}u^{1/5}}{3}du$ 
(E) none of these

20. Approximate the area under the graph of  $f(x) = 2^{x-1} + 3$  and above the x-axis from x = -3 to x = 5 with rectangles, using left endpoints, with n = 4.

The approximate area is

- (A) less than 33 (B) between 33 and 38 (C) between 38 and 43
- (D) between 43 and 48 (E) more than 48

- 21. Find the area under the graph of  $f(x) = 3x^2 + 1$  and above the x-axis from x = -1 to x = 3. The area is
  - (A) between 15 and 20 (B) between 20 and 25 (C) between 25 and 30
  - (**D**) between 30 and 35 (**E**) between 35 and 40

22. Find the consumers' surplus if the demand function for a particular sandwich cookie is given by

 $D(q) = 5000 - 0.02q^2 - 0.002q^3.$ 

Assume the supply and demand are in equilibrium at q = 80. The consumers' surplus is

- (A) between \$67,000 and \$67,500 (B) between \$67,500 and \$68,000
- (C) between \$68,000 and \$68,500 (D) between \$68,500 and \$69,000
- (E) between \$69,000 and \$69,500

23. The rate of a continuous money flow starts at \$800 and increases exponentially at 5% per year for 7 years. Find the accumulated amount of money flow at the end of 7 years if interest earned is 2% compounded continuously.

The accumulated amount is

- (A) less than \$6400 (B) between \$6400 and \$6700
- (C) between \$6700 and \$7000
- (D) between \$7000 and \$7300

(E) more than \$7300

24. May Klingman invests \$23,000 in an account paying 5.4% per year, compounded monthly. How many years are required for the compound amount to at least double?

The time it takes for the compound amount to at least double is

- (A) between 4 and 6 years (B) between 6 and 8 years (C) between 8 and 10 years
- (D) between 10 and 12 years (E) between 12 and 14 years

SHORT ANSWER: Be sure to show all work, define all variables, label all units. Graphs should be drawn in appropriate windows and should include axes that are labled and scaled. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit.

1. Suppose a cost-benefit model is given by

$$y = \frac{8.7x}{100 - x}$$

where x is a number of percent and y is the cost, in thousands of dollars, of removing x percent of a given pollutant.

(a) Find the cost of removing 90% of the given pollutant.

(b) Is it possible, according to this function, to remove all the pollutant? (Justify your answer.)

(c) Graph the function.

- 2. A local club is arranging a charter flight to Hawaii. The cost of the trip is \$840 each for 65 passengers, with a refund of \$8 per passenger for each passenger in excess of 65.
  - (a) Find the number of passengers that will maximize the revenue received from the flight. (Note: to receive full credit, you must demonstrate that your answer is an absolute maximum.)

(b) What is the maximum revenue?

3. The supply function for oil is given (in dollars) by S(q), and the demand function is given (in dollars) by D(q):

$$S(q) = q^2 + 9q;$$
  $D(q) = 986 - 15q - q^2.$ 

(a) Graph the supply and demand curves on the same axes.

(b) Find the point at which supply and demand are in equilibrium.

(c) Find the consumers' surplus.

(d) Find the producers' surplus.

## Formulas You Might Find Useful

Ι	=	Prt	Α	=	$P\left(1+\frac{r}{m}\right)^{mt}$
A	=	$Pe^{rt}$	$r_E$	=	$\left(1+\frac{r}{m}\right)^m - 1$
$r_E$	=	$e^r - 1$	f'(x)	=	$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx}\left[f(g(x))\right]$	=	$f'(g(x))\cdot g'(x)$	$\frac{d}{dx}\left[u(x)\cdot v(x)\right]$	=	$u(x) \cdot v'(x) + v(x) \cdot u'(x)$
$\frac{d}{dx} \left[ \frac{u(x)}{v(x)} \right]$	=	$\frac{v(x)\cdot u'(x)-u(x)\cdot v'(x)}{[v(x)]^2}$	$\frac{d}{dx}\left[a^{x}\right]$	=	$(\ln a)a^x$
$\frac{d}{dx}\left[\log_a(x)\right]$	=	$\frac{1}{(\ln a)x}$	E	=	$-rac{p}{q}\cdot rac{dq}{dp}$
q	=	$\sqrt{\frac{2fM}{k}}$	$\int x^n dx$	=	$\frac{x^{n+1}}{n+1} + C$
$\int a^{kx} dx$	=	$\frac{a^{kx}}{k(\ln(a))} + C$	$\int x^{-1} dx$	=	$\ln  x  + C$
C.S.	=	$\int_0^{q_0} \left( D(q) - p_0 \right) dq$	<i>P.S.</i>	=	$\int_0^{q_0} \left( p_0 - S(q) \right) dq$
Р	=	$\int_0^T f(t)e^{-rt}dt$	A	=	$e^{rT} \int_0^T f(t) e^{-rt} dt$