Math 116 - Spring 2014
Final Exam Solutions- Version A

MULTIPLE CHOICE: Indicate your answer on the Scantron sheet provided and circle your answer in this test booklet. No partial credit will be given in this section.

#1 A	#7 E	#13 D	#19 A
#2 B	#8 A	#14 B	#20 B
#3 C	#9 A	#15 C	#21 D
#4 D	#10 C	#16 D	#22 C
#5 B	#11 B	#17 C	#23 D
#6 E	#12 A	#18 E	#24 E

SHORT ANSWER: Be sure to show all work, define all variables, label all units. Partial credit may be given for partially correct answers. Answers without justification will not receive full credit.

1. Suppose a cost-benefit model is given by

$$y = \frac{8.7x}{100 - x}$$

where x is a number of percent and y is the cost, in thousands of dollars, of removing x percent of a given pollutant.

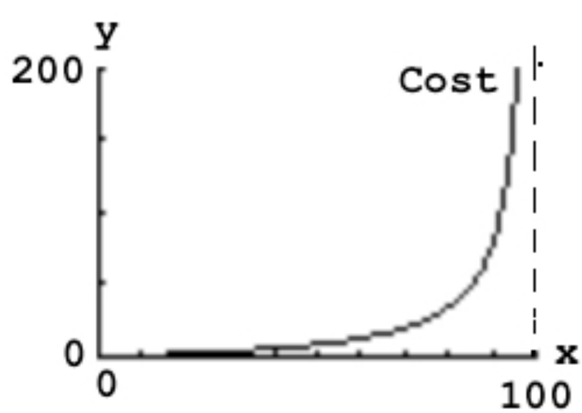
(a) *(3 points)* Find the cost of removing 90% of the given pollutant.

$y = \dfrac{8.7(90)}{100 - (90)}$
$y = \$78.3$ thousand, or
$y = \$78,300$

(b) *(3 points)* Is it possible, according to this function, to remove all the pollutant? (Justify your answer.)

No;
If $x = 100$, then the cost-benefit function is undefined. Or,
There is a vertical asymptote at $x = 100$.

(c) *(4 points)* Graph the function.



2. A local club is arranging a charter flight to Hawaii. The cost of the trip is \$840 each for 65 passengers, with a refund of \$8 per passenger for each passenger in excess of 65.

   (a) *(8 points)* Find the number of passengers that will maximize the revenue received from the flight.

   Let x be the number of passengers
   in excess of 65
   \# of Passengers: $(65 + x)$
   Revenue from Each Pass.: $(840 - 8x)$
   Total Revenue: $R(x) = (65 + x)(840 - 8x)$
   or $R(x) = -8x^2 + 320x + 54600$.

   We will find the maximum by
   1st: finding all critical points
   2nd: testing the critical points and endpoints.

   We find critical points by finding $R'(x)$ and
   determining where it is zero or undefined.
   $R'(x) = -16x + 320$
   $R'(x)$ is never undefined, and is zero
   when $x = 20$.

   From the context, we see that a maximum
   cannot occur if x is less than zero,
   nor if x is more than 105.

   Now we test the critical point and endpoints:
   $R(0) = 54,600$, $R(20) = 57,800$, $R(105) = 0$
   so the absolute maximum occurs at $x = 20$,
   which corresponds to 85 passengers.

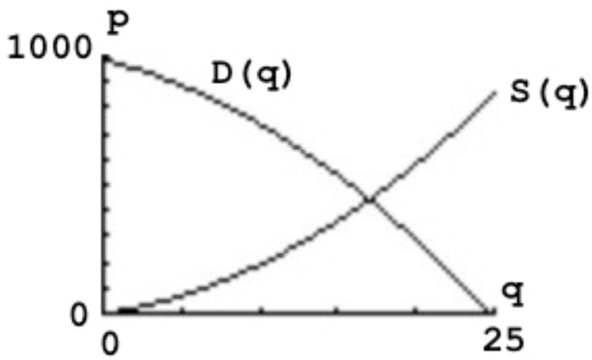
   (b) *(2 points)* What is the maximum revenue?

   The maximum revenue is $\$57,800$.

3. The supply function for oil is given (in dollars) by $S(q)$, and the demand function is given (in dollars) by $D(q)$:

$$S(q) = q^2 + 9q; \qquad D(q) = 986 - 15q - q^2.$$

(a) *(2 points)* Graph the supply and demand curves on the same axes.



(b) *(2 points)* Find the point at which supply and demand are in equilibrium.

Supply and demand are in equilibrium when
$S(q) = D(q)$
$q^2 + 9q = 986 - 15q - q^2$
$2q^2 + 24q - 986 = 0$
$2(q - 17)(q + 29) = 0$
$q = 17$ or $q = -29$
Since $q = -29$ makes no sense in context, and $D(17) = S(17) = 442$, our equilibrium solution is $q = 17$, $p = \$442$.

(c) *(3 points)* Find the consumers' surplus.

$$\begin{aligned} C.S. &= \int_0^{q_0} (D(q) - p_0)\, dq \\ &= \int_0^{17} \big((986 - 15q - q^2) - 442\big)\, dq = \$5442.83 \end{aligned}$$

(d) *(3 points)* Find the producers' surplus.

$$\begin{aligned} P.S. &= \int_0^{q_0} (p_0 - S(q))\, dq \\ &= \int_0^{17} \big(442 - (q^2 + 9q)\big)\, dq = \$4575.83 \end{aligned}$$