Section 2.1: How do we measure speed?

It turns out that it is not very simple to give a precise definition of speed. Intuitively, we understand very easily what is necessary to give an idea of how fast we are traveling. For example, if I told you that Mesa Arizona was 120 miles away from Tucson (it's not exactly that far) and that I wanted to make the trip from Tucson to Mesa in two hours, you would almost immediately tell me that I needed to drive 60 miles per hour, on average, in order to make the journey in that time.

What I described above is an example of *average speed*. This is something we have understood for a very long time:

$$\text{AVERAGE SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

There probably isn't a more well-known equation in the universe, other than Einstein's $E = mc^2$. But what if we wanted to know what an object's speed was at a given *instant* of time? This problem is much more difficult. In fact, this problem was of great concern to many philosophers going way back in time (try a Google search for Zeno's paradox). But before we get too involved in this, let us first make a distinction between *speed* and *velocity*

Velocity vs. Speed

Concept: What is the difference between velocity and speed?

If $s(t)$ is the position of an object at time t, then the *average velocity* of the object over the interval $a \leq t \leq b$ is

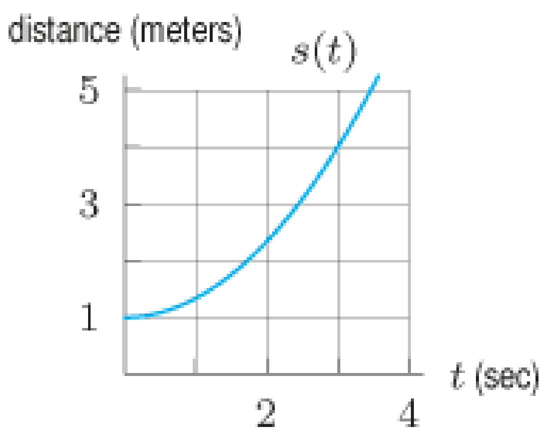
$$\text{Average Velocity} = \frac{\Delta s}{\Delta t} = \frac{s(b) - s(a)}{b - a}.$$

Examples:

1. The table gives the position of a particle moving along the x-axis as a function of time in seconds, where x is in meters. What is the average velocity of the particle from $t = 0$ to $t = 4$?

t	0	2	4	6	8
$x(t)$	-2	4	-6	-18	-14

2. The figure below shows a particle's distance from a point. What is the particle's average velocity from $t = 0$ to $t = 3$?



3. At time t seconds, a particle's distance, in micrometers (μm), from a point is given by $s(t) = e^t - 1$. What is the average velocity of the particle from $t = 2$ to $t = 4$?

Instantaneous Velocity

So how is one to define isntantaneous velocity? As we have discovered, motion doesn't exist in a frozen instant in time. The following example should illustrate how we have overcome this difficulty.

4. In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s = 3t^2$.

   (a) Find the average velocity between $t = 1$ and $t = 1 + h$ if

      (i) $h = 0.1$

      (ii) $h = 0.01$

(iii) $h = 0.001$.

(b) Use your answers from part (a) to estimate the instantaneous velocity at $t = 1$.

INSTANTANEOUS VELOCITY: Let $s(t)$ be the position at time t. Then the *instantaneous velocity* at time $t = a$ is defined as

$$\lim_{h \to 0} \frac{s(a+h) - s(a)}{h}.$$

5. Suppose that the position of a particle after t seconds is given by $s(t) = t^2$, where s is measured in meters. Find the instantaneous velocity of the particle at $t = 2$.

Instantaneous Velocity as Slope

The *average velocity* of a particle from $t = a$ to $t = b$ can be visualized as the slope of the line connecting $(a, s(a))$ to $(b, s(b))$: