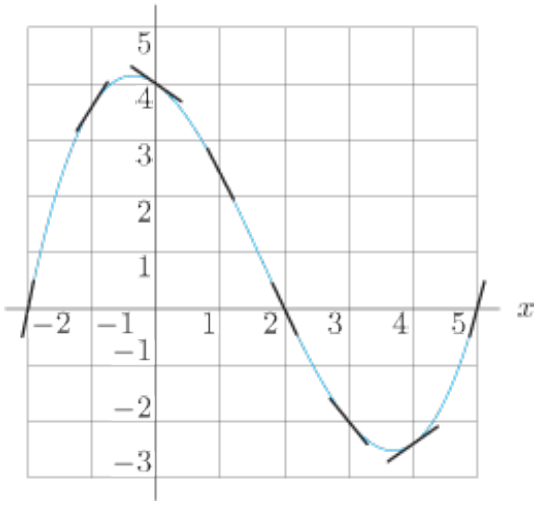
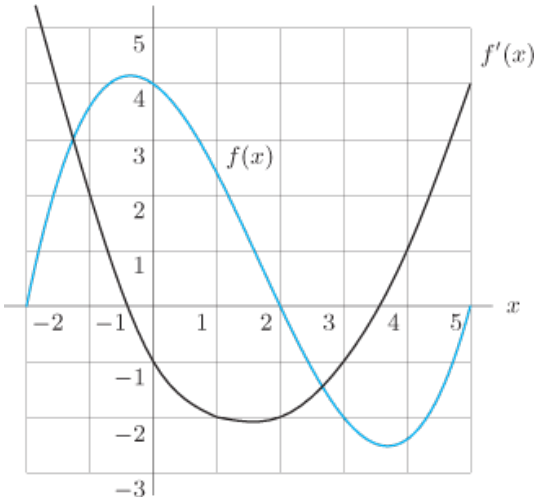
Section 2.3: The Derivative Function

Given a function $f(x)$, we can evaluate the slope of the function at any point in the domain:



If we plot the values of the slope of $f(x)$ for every relative value of x in the domain of f, we obtain a new function:



For any function f, we define the *derivative function*, f', by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Given a function f, what does the derivative, f', tell us about f?

Concepts:

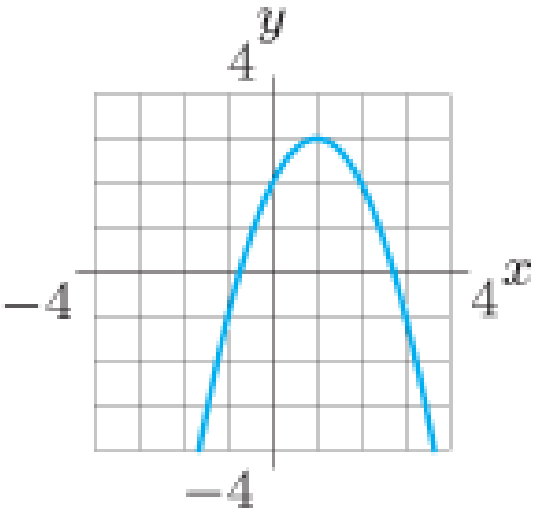
(i) If $f' > 0$ on an interval, is f increasing or decreasing on that interval?

(ii) If $f' < 0$ on an interval, is f increasing or decreasing on that interval?

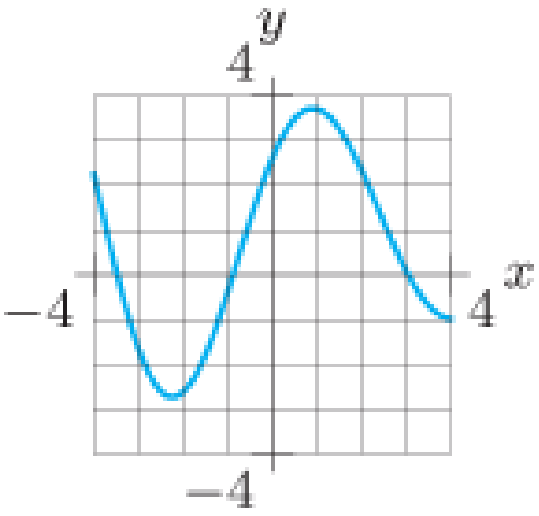
(iii) If $f'(a) = 0$, what can we say about the graph of $f(x)$ at $x = a$?

Examples:

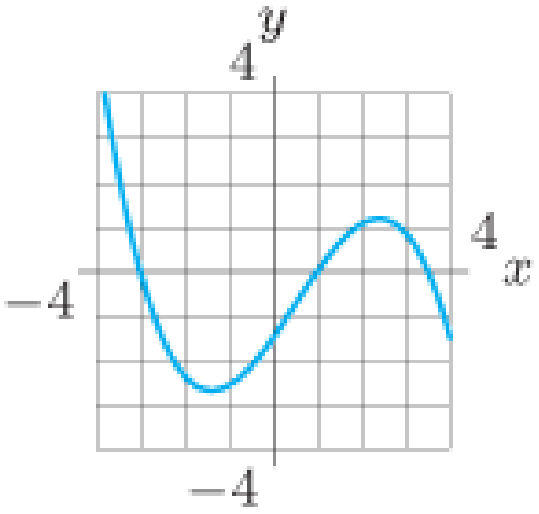
1. Graph the derivative of the given function



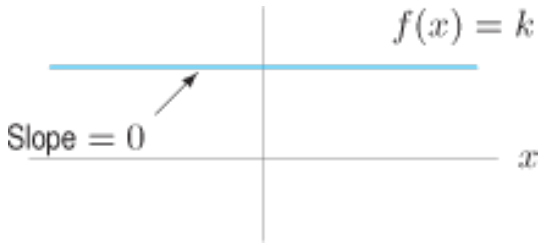
2. Graph the derivative of the given function



3. Graph the derivative of the given function



Derivative of a Constant function: Using the image below, write down a general formula for the derivative of any function $f(x) = k$, where k is an arbitrary constant.



Derivative of a Linear Functions: Write down a general formula for the derivative of any linear function $f(x) = mx + b$, where m and b are arbitrary constants.

Examples:

4. Use the limit definition of the derivative to find the derivative function for

$$g(x) = \frac{1}{x}$$

5. Use the limit definition of the derivative to find the derivative function for

$$h(t) = \sqrt{t}$$