Section 3.1: Powers and Polynomials

Question: If $f'(a) = 2$, then what is the slope of the tangent line to the function $g(x) = 3f(x)$ at $x = a$?

THEOREM 3.1: DERIVATIVE OF A CONSTANT MULTIPLE: If f is differentiable and c is a constant, then

$$\frac{d}{dx}\left[cf(x)\right] = cf'(x).$$

Question: If $f'(a) = 2$ and if $g'(a) = 4$, then what is the slope of the tangent line to the function $h(x) = f(x) + g(x)$ at $x = a$?

THEOREM 3.2: DERIVATIVE OF A SUM AND DIFFERENCE: If f and g are differentiable, then

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x).$$

Back in chapter two, we recognized a pattern when taking derivatives of functions of the form $f(x) = x^n$, where n is any real number. This led us to the following conclusion:

THE POWER RULE: For any constant real number n,

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}.$$

Examples:

1. Find the derivative of the following functions.

   (a) $y = x^2 + 5x + 7$

   (b) $f(z) = -\dfrac{1}{z^{6.1}}$

   (c) $h(\theta) = \dfrac{1}{\sqrt[3]{\theta}}$

   (d) $h(x) = \ln e^{ax}$

   (e) $y = z^2 + \dfrac{1}{2z}$

(f) $h(w) = -2w^{-3} + 3\sqrt{w}$

(g) $y = 3t^5 - 5\sqrt{t} + \dfrac{7}{t}$

(h) $y = \sqrt{x}\,(x+1)$

(i) $y = \dfrac{x^2+1}{x}$

2. The graph of $y = x^3 - 9x^2 - 16x + 1$ has a slope of 5 at two points. Find the coordinates of the points.

3. Over what intervals is the function $f(x) = x^4 - 4x^3$ both decreasing and concave up?

4. Find an equation of the tangent line to $f(x)$ at $x = 2$ if
$$f(x) = \frac{x^3}{2} - \frac{4}{3x}.$$

5. At time t seconds after it is thrown into the air, a tomato is at a height of $f(t) = -4.9t^2 + 25t + 3$ meters.

   (a) What is the instantaneous velocity of the tomato at $t = 2$?

   (b) What is the acceleration at $t = 2$?

   (c) How high does the tomato go?

   (d) How long is the tomato in the air?